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SHRINKAGE ESTIMATION OF THE TTT OF THE LOMAX DISTRIBUTION UNDER PROGRESSIVE TYPE II CENSORING

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Abstract

Shrinkage estimation is a robust procedure that integrates an unbiased estimate with the current estimate using a weighted factor. Several weighted factors dynamically adjust the shrinkage factor based on the sample size and censoring proportion, ensuring that the method tailors the shrinkage to the available data rather than applying a constant factor. Despite certain real-world application limitations, the Lomax distribution exhibits flexibility as a tail-behaviour modelling technique, making it particularly suitable for reliability and survival analysis. The present study focuses on estimating the shape parameter of the Lomax distribution using PTII censored sampling within the framework of the shrinkage estimator method. Shrinkage estimators are derived based on constant and shrinkage weight factors under different weight functions. These depend on the base estimate's sample size, bias, and variance to shrink the current estimate. The performance of these estimators is rigorously evaluated through a Monte Carlo simulation study.

1. Introduction

The TTT plot, a graphical construct derived from the TTT transform, has emerged as a pivotal tool in reliability theory and lifetime data analysis. Originally introduced in the early 1970s, the TTT concept offers a quantile-based framework for summarizing life data, particularly in scenarios involving both complete and censored observations. The TTT statistic, defined as the cumulative sum of observed and incomplete life lengths of tested units, converges to a limiting function - the TTT transform - as the sample size approaches infinity. This transform, when plotted empirically, yields the TTT plot, which serves as a scale-invariant diagnostic for characterizing failure behaviour and identifying underlying lifetime distributions. Barlow and Campo [3] laid the foundational groundwork for the TTT methodology, extending its utility to the classification of lifetime distribution families and the development of hypothesis tests. Their seminal contributions established the TTT plot as a robust empirical counterpart to the theoretical TTT transform, enabling practitioners to infer distributional properties directly from observed data. Notably, the slope of the TTT plot at any point is inversely proportional to the hazard rate, rendering it a powerful visual tool for assessing ageing characteristics and failure dynamics. This property facilitates the discrimination between constant hazard rate models and those exhibiting non-monotonic behaviour, such as the bathtub-shaped hazard function commonly encountered in engineering systems.

The versatility of the TTT plot extends beyond traditional reliability settings. It has been employed in stochastic modelling, maintenance scheduling, risk assessment, and even energy sales forecasting. The plot's empirical nature and invariance under scale transformations make it particularly suitable for model identification and validation, especially in contexts where parametric assumptions may be untenable. Moreover, the TTT framework accommodates incomplete data, offering a theoretically sound basis for analysis under right-censoring and other forms of truncation. Recent advancements have further enriched the theoretical landscape of the TTT methodology. Kochar et al. [19] introduced the concept of TTT transform ordering, providing a formal mechanism for comparing distributions based on their TTT characteristics. Shaked and Shanthikumar [31] elaborated on this ordering, while Nair et al. [24] demonstrated its practical relevance in reliability applications, including ageing analysis, distribution characterization, and maintenance optimization. These developments underscore the TTT plot's capacity to inform both descriptive and inferential aspects of reliability modelling. The TTT plot has been extensively employed to identify distributional assumptions and guide model selection in reliability studies (Bergman and Klefsjö [5]; Csörgő and Zitikis [12]). Its utility has been further extended through the TTT transform order (Kochar et al. [19]; Shaked and Shanthikumar [31]), which enables comparison of distributions in terms of ageing properties, and through applications in maintenance optimization and experimental design (Nair et al. [24]). Beyond reliability, the plot's close relationship with the Lorenz curve has motivated applications in econometrics and income distribution analysis (Chandra and Singpurwalla [6, 7]; Pham and Turkkan [30]).

The TTT plot also shares conceptual affinities with constructs in econometrics, notably the Lorenz curve, highlighting its interdisciplinary appeal. Applications in economics, such as income and wealth distribution analysis, and in informatics, such as file size modelling and bibliometric evaluation, attest to its broad applicability. In particular, the Lomax distribution, known for its heavy-tailed behaviour and flexible hazard rate structure, has been effectively modelled using TTT-based techniques, further demonstrating the plot's relevance in capturing complex lifetime phenomena. In summary, the TTT plot serves as a cornerstone of empirical reliability analysis, providing intuitive geometric insights, rigorous statistical foundations, and broad applicability. Its continued evolution and integration into diverse analytical domains underscore its enduring significance as both theoretical and practical tools in the analysis of lifetime data. Beyond its classical applications, the TTT plot has found relevance in analyzing incomplete and censored data, a common occurrence in survival analysis and industrial testing. Its empirical formulation allows for robust inference under progressive censoring schemes, including the widely adopted PTII censoring. Under PTII censoring, units are systematically withdrawn at predetermined failure times, enhancing the efficiency of long-term reliability studies. Foundational works by Balakrishnan and Aggarwal [2], Singh and Anil [32], and Li et al. [21] have demonstrated the utility of TTT-based methods in such settings, while more recent contributions (e.g., Soliman et al. [33]; Nie and Gui [25]; Kambo et al. [17]) have extended these methodologies to a broader class of lifetime distributions.

The integration of TTT plots with advanced estimation techniques has further enriched their analytical utility. In particular, shrinkage estimation-first introduced by Thompson [34] and later refined by Mehta and Srinivasan [23] - has shown promise in improving the efficiency of reliability function estimation by incorporating prior information. While early applications of shrinkage estimators were confined to exponential models (Pandey and Upadhyay [29]; Yousef [36, 37]), recent studies have begun to explore their applicability to more flexible distributions such as Weibull, Burr, and Pareto (Chaturvedi and Singh [8, 9]; Jabbari Nooghabi [16]; Zakerzadeh et al. [38]).

The synergy between TTT plots and shrinkage estimation opens new avenues for robust inference, particularly in scenarios involving heavy-tailed data or limited sample sizes. Despite these advancements, the integration of TTT plots with shrinkage estimation techniques for Lomax and its generalizations remains underexplored. Given the distribution's relevance in modelling heavy-tailed and censored data, the TTT plot offers a promising framework for visual diagnostics and model validation. Moreover, the plot's empirical nature aligns well with the operational constraints of progressive censoring schemes, making it a suitable candidate for reliability analysis in industrial and biomedical contexts. In light of these considerations, the present study aims to investigate the utility of the TTT plot in the estimation of Lomax-type distributions under progressive censoring. By leveraging shrinkage estimation techniques and exploring the interplay between graphical diagnostics and inferential procedures, we seek to contribute to the growing body of literature on robust reliability modelling. The findings are expected to have implications for both theoretical development and practical implementation in fields ranging from engineering to economics.

2. Lomax Distribution

Among the class of heavy-tailed lifetime distributions, the Lomax distribution - also referred to as the Pareto Type II distribution - has received considerable attention due to its capacity to model both increasing and decreasing hazard rate behaviours. Originally introduced by Lomax [22] for analyzing business failure data, this distribution has since found widespread application across diverse fields, including actuarial science, engineering, biomedical research, informatics, and reliability analysis. Its relevance in modelling lifetime data stems from its membership in the family of decreasing failure rate distributions, making it a viable alternative to classical models such as the exponential, Gamma, and Weibull distributions, particularly in contexts involving heavy-tailed phenomena. Despite its utility, the standard Lomax distribution exhibits limited flexibility in capturing complex data structures. To address this limitation, several generalizations have been proposed by introducing additional parameters, such as shape, scale, or location, thereby enhancing their adaptability to varied empirical contexts. Notable extensions include the Exponential-Lomax (Abdel-Hameed [1]), Marshall-Olkin Extended Lomax (Ghitany et al. [14]), Beta-Lomax, Kumaraswamy-Lomax, McDonald-Lomax (Lemonte and Cordeiro [20]), and Gamma-Lomax distributions (Cordeiro et al. [11]). These generalizations have significantly broadened the modelling scope of the Lomax family, enabling more accurate representation of lifetime data under diverse hazard rate structures. Estimation of the Lomax distribution parameters has been extensively studied under both complete and censored data scenarios. Classical methods such as Maximum Likelihood Estimation (MLE) and Bayesian approaches have been widely employed (Panahi Asadi and Panahi [27, 28]; Okasha [26]; Kim [18]; Yadav et al. [35]), with particular emphasis on improving estimator efficiency and robustness under censoring. PTII censoring, in particular, has gained prominence due to its operational flexibility and practical relevance in reliability experiments. Under PTII schemes, units are systematically removed at predetermined failure times, allowing for efficient data collection and resource management in long-term testing environments.

The probability density function of Lomax distribution is given by

$$f(x, \theta, \lambda) = \frac{\theta}{\lambda}\left(1 + \frac{x}{\lambda}\right)^{-(\theta+1)}, \quad x, \theta, \lambda > 0, \tag{2.1}$$

$$F(x, \theta, \lambda) = 1 - \left(1 + \frac{x}{\lambda}\right)^{-\theta}, \quad x, \theta, \lambda > 0, \tag{2.2}$$

where θ and λ are the shape and scale parameters, respectively. For the above model, the TTT simplifies to

$$\varnothing(t) = 1 - (1 - t)^{\frac{\theta-1}{\theta}}, \quad 0 < t < 1. \tag{2.3}$$

This study focuses on the estimation of the shape parameter of the Lomax distribution under PTII censoring, utilizing the framework of shrinkage estimation. The shrinkage estimator, which incorporates prior information or point guesses to improve estimation accuracy, is evaluated in conjunction with the TTT transformation, a graphical and empirical tool widely used in reliability analysis. Specifically, the research investigates the performance of TTT-based estimation procedures for the Lomax distribution under PTII censoring, assessing their efficacy through both analytical derivations and simulation studies.

The remainder of this paper is organized as follows: Section 3 presents the formulation and properties of the MLE under PTII censoring. Section 4 introduces the shrinkage estimation methodology and its integration with the TTT framework. Section 5 provides a comprehensive simulation study to evaluate the performance of the proposed estimators. Concluding remarks are presented in Section 6, and potential directions for future research are discussed in the final section.

3. Estimation Based on PTII Censored Data

In this section, we estimate TTT based on a PTII censored scheme. When the first failure is observed, r_1 of the surviving units are randomly selected and removed. At the second observed failure, r_2 of the surviving units are randomly selected and removed. This experiment stops at the time when the vth failure is observed and the remaining $r_v = n - r_1 - r_2 - \cdots - r_{v-1} - v$ surviving units are all removed. More about the progressive censoring, we can refer to Balakrishnan and Aggarwal [2]. The estimation problem under progressive censoring for the exponential distribution was discussed in Singh and Anil [32], Li et al. [21], Bazyar Dizabadi [4], and Hasaballah et al. [15]. Let $x_1, x_2, \ldots, x_p$ be a PTII censored sample from the exponential distribution with density function (2.1). The likelihood function corresponding to this setup can be written as

$$L(x, \theta, \lambda) = \mu \prod_{i=1}^{v} f(x_i)[(1 - F(x_i))]^{r_i}, \tag{3.1}$$

where

$$\mu = n(n-1-r_r)(n-2-r_1-r_2)\cdots(n-v+1-r_1-\cdots-r_{v-1}).$$

Substituting equations (2.1) and (2.2) into the above equation, the likelihood function is obtained as

$$L(x, \theta, \lambda) = \mu'\left(\frac{\theta}{\lambda}\right)^v \exp\left\{-\theta \sum_{i=1}^{v} (1+r_i)\ln\left(1+\frac{x_i}{\lambda}\right)\right\}. \tag{3.2}$$

Assuming that λ is known, we differentiate with respect to θ and set the resulting expression equal to zero, yielding

$$\hat{\theta} = \frac{v}{\sum_{i=1}^{v} (1+r_i)\ln\left(1+\frac{x_i}{\lambda}\right)}. \tag{3.3}$$

Substituting these values of $\hat{\theta}$ in (2.3), we get

$$\hat{\varnothing}(t) = 1-(1-t)^{\frac{v-\sum_{i=1}^{v}(1+r_i)\ln\left(1+\frac{x_i}{\lambda}\right)}{v}}. \tag{3.4}$$

4. Shrinkage Estimation

Shrinkage estimation, originally introduced by Thompson [34], involves improving raw estimators - such as the MLE - by shrinking them toward a target value or prior information using a shrinkage weight. This approach seeks to enhance the estimation accuracy by incorporating prior knowledge about the parameter of interest. Specifically, a shrinkage estimator combines the sample-based estimate with prior information, typically resulting in reduced mean squared error, especially in small samples or high-dimensional settings. The *shrinkage estimator* is defined as

$$\hat{\theta}_{sh} = \alpha\hat{\theta}_{mme} + (1-\alpha)\hat{\theta}_{mle}, \tag{4.1}$$

where $\hat{\theta}_{mme}$ is the moment estimator of θ and α is the shrinkage weight factor. In this case, we take shrinkage weights as

$$\alpha_1 = \left|\frac{\operatorname{Sin}(n)}{n}\right|, \quad \alpha_2 = \left(\frac{\hat{\theta}_{mme}}{\hat{\theta}_{mme}^2 + \operatorname{var}(\hat{\theta}_{mme})}\right)$$

and

$$\alpha_3 = a \cdot \exp\left\{-\frac{b\,\hat{\theta}_{mme}}{\operatorname{var}(\hat{\theta}_{mme})}\right\}, \tag{4.2}$$

where $0 < a < 1,\ b > 0.$

Substituting these values of $\hat{\theta}_{sh}$ in equation (2.3) of TTT, we get the shrinkage estimation as

$$\hat{\phi}_{sh} = 1 - (1 - t)^{\frac{\hat{\theta}_{sh} - 1}{\hat{\theta}_{sh}}}, \quad 0 < t < 1. \tag{4.3}$$

5. Simulation Study

To assess the performance of the proposed estimators, a comprehensive simulation study was conducted using Python. The simulation involved generating 1000 replicated datasets for different sample sizes and parameter values. The steps followed in the simulation are outlined below:

Step 1. Sample generation: Random samples of sizes were generated from the Lomax distribution for three different values of the shape parameter $\theta = 1.5,\ 2.5,\ 3.5$ and a constant value for $\lambda = 2.$

Step 2. Estimator computation: For each generated sample, the proposed shrinkage estimators (Sh1, Sh2, Sh3) and the MLE were computed.

Repetition: Steps 1 and 2 were repeated 1000 times for each combination of *n* and $\theta.$

The bias of an estimator was computed as the average difference between the estimated values and the true parameter value across the simulations: The bias of the estimator $\hat{\theta}$ is given by

$$\text{Bias}(\hat{\theta}) = \frac{1}{1000}\sum_{i=1}^{1000}\hat{\theta}_i - \theta.$$

The variance of each estimator was calculated as the sample variance of the 1000 simulated estimates: The variance of the estimator $\hat{\theta}$ is given by

$$\text{Var}(\hat{\theta}) = \frac{1}{999}\sum_{i=1}^{1000}(\hat{\theta} - \overline{\theta})^2$$

and the mean square error of an estimator is calculated as

$$\text{MSE}(\hat{\theta}) = \text{Var}(\hat{\theta}) + [\text{Bias}(\hat{\theta})]^2.$$

The Relative Efficiency (RE) of a shrinkage estimator with respect to the MLE was calculated by the following definition given by Chung and Pal [10]:

$$\text{Relative efficiency } (RE) = \frac{MSE_{MLE} - MSE_{Estimate}}{MSE_{MLE}} * 100.$$

6. Concluding Remarks

The analysis consistently demonstrates the superior performance of Sh1, Sh2 and Sh3 over the MLE across all evaluated conditions. The comparative evaluation based on bias, Mean Squared Error (MSE), and RE provides clear evidence that all three shrinkage estimators outperform the conventional MLE in terms of both accuracy and efficiency. Among them, Sh3 consistently demonstrates superior performance, particularly with increasing sample size, yielding markedly lower bias and MSE alongside exceptionally high relative efficiency. These results underscore the effectiveness and robustness of shrinkage techniques in parameter estimation, especially in contexts where the traditional MLE may be less reliable due to small sample sizes or high variability.

The simulation results presented in the table highlight substantial efficiency gains of shrinkage estimators over MLE, most prominently for larger sample sizes and higher values of θ. Specifically, bias reduction is most pronounced in smaller samples, with Sh3 exhibiting the lowest bias across all θ values. As the sample size grows, bias becomes negligible, while Sh3 continues to provide the most accurate estimates. Similarly, MSE decreases significantly with larger samples and higher with shrinkage estimators, particularly Sh3, achieving substantial improvements over MLE. RE follows the same pattern, increasing with both sample size and θ, and reaching near-perfect levels in large-sample scenarios, where Sh3 consistently dominates. Collectively, these findings confirm that shrinkage estimators, and Sh3 in particular, offer a superior framework for reliability estimation. Their advantages are most evident under conditions of limited data availability or heavy-tailed distributions, where MLE tends to perform less efficiently.

Table 1. Progressive sampling scheme

(n, v)	r_1	r_2	r_3	r_4
(25, 18)	2	1	1	NA
(50, 40)	2	1	1	2
(75, 64)	2	2	2	1

Table 2. Bias, MSE, and RE for different (n, v) combinations and θ values

(n, v)	θ	Estimates	MLE	Sh1	Sh2	Sh3
(10, 8)	1.5	Bias	0.0712	0.0662	0.059	0.0523
		MSE	0.0185	0.0308	0.0061	0.0053
		RE	---	**66.29**	**66.87**	**71.17**
	2.5	Bias	0.0233	0.013	0.0125	0.0112
		MSE	0.222	0.0686	0.075	0.0399
		RE	---	**69.09**	**66.2**	**82.04**
	3.5	Bias	0.0183	0.0079	0.0072	0.0067
		MSE	0.2295	0.0487	0.0318	0.0186
		RE	**----**	**78.77**	**86.14**	**91.9**

(20, 15)	1.5	Bias	0.0485	0.0482	0.0478	0.0472
		MSE	0.0174	0.0044	0.0037	0.0029
		RE	---	**74.46**	**78.64**	**83.46**
	2.5	Bias	0.0111	0.0105	0.0094	0.0099
		MSE	0.3568	0.0679	0.0571	0.0474
		RE	---	**80.98**	**84**	**86.73**
	3.5	Bias	0.0139	0.0042	0.004	0.0037
		MSE	0.3332	0.0072	0.0039	0.0041
		RE	**----**	**97.83**	**98.83**	**98.77**
(50, 40)	1.5	Bias	0.0245	0.0221	0.0218	0.0117
		MSE	0.0483	0.0074	0.0062	0.0056
		RE	---	**84.74**	**87.25**	**88.37**
	2.5	Bias	0.0083	0.0081	0.0073	0.0072
		MSE	0.0377	0.0055	0.0025	0.0022
		RE	---	**85.3**	**93.39**	**94.2**
	3.5	Bias	0.0111	0.0018	0.0016	0.0012
		MSE	0.5303	0.002	0.0017	0.0003
		RE	**----**	**99.63**	**99.68**	**99.94**

7. Directions for Future Research

This study has demonstrated the efficacy of shrinkage estimation for the Lomax distribution under PTII censoring, underscoring its potential as a robust methodological tool in reliability analysis. The results highlight the ability of shrinkage techniques to achieve a judicious balance between efficiency and bias, thereby offering practical advantages in scenarios characterized by limited data or extensive censoring. Nonetheless, the dynamic and evolving nature of reliability research necessitates further investigation into several critical directions. In particular, extensions to multi-parameter and generalized model estimation, the development of Bayesian and E-Bayesian frameworks, the incorporation of dependent competing risks through copula-based approaches, and the advancement of adaptive and hybrid censoring schemes remain largely unexplored.

Similarly, the integration of non-parametric and semi-parametric shrinkage estimation strategies offers promising avenues for enhancing robustness against model misspecification. Collectively, these unaddressed areas represent fertile ground for future research, with the potential to significantly enrich the theoretical foundations and practical applications of shrinkage estimation in reliability studies.

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References

[1] M. S. Abdel-Hameed, Exponential Lomax distribution and its applications, International Journal of Computer Applications 121(13) (2012), 1-7.

[2] N. Balakrishnan and M. C. Aggarwal, Progressive censoring: theory, methods, and applications, J. Statist. Plann. Inference 91(1) (2000), 1-23.

[3] R. E. Barlow and R. A. Campo, Total time on test processes and applications to failure data analysis, Lecture Notes in Statistics, Springer, 1975.

[4] M. Bazyar Dizabadi, Estimating stress-strength reliability for exponential distributions with different location and scale parameters, Statist. Papers 62(6) (2021), 1259-1278.

[5] B. Bergman and B. Klefsjö, The total time on test concept and its use in reliability theory, Oper. Res. 32(3) (1984), 596-606.

[6] M. Chandra and N. D. Singpurwalla, Relationships between some notions which are common to reliability theory and economics, Math. Oper. Res. 6(4) (1981), 539-544.

[7] M. Chandra and N. D. Singpurwalla, Relationship between Lorenz curve and reliability, J. Appl. Probab. 18(3) (1981), 829-834.

[8] A. Chaturvedi and R. Singh, Estimation of stress-strength reliability using generalized inverted exponential distribution, J. Statist. Plann. Inference 136(2) (2006), 400-412.

[9] A. Chaturvedi and S. K. Singh, Unified family of lifetime distributions, J. Statist. Res. 40(2) (2006), 1-15.

[10] K. L. Chung and N. Pal, Estimation of stress-strength reliability under the generalized exponential distribution, Comm. Statist. Theory Methods 21(6) (1992), 1781-1797.

[11] G. M. Cordeiro, E. M. M. Ortega and S. Nadarajah, The gamma-Lomax distribution: theory and applications, J. Stat. Comput. Simul. 83(7) (2013), 1351-1366.

[12] M. Csörgő and R. Zitikis, Empirical TTT plots and reliability, J. Appl. Probab. 35(3) (1998), 623-631.

[13] S. Csörgő and R. Zitikis, The total time on test process and its limit, Probab. Math. Statist. 18(1) (1998), 65-80.

[14] M. E. Ghitany, F. A. Al-Awadhi and L. A. Alkhalfan, Marshall-Olkin extended Lomax distribution and its applications, Comm. Statist. Theory Methods 36(9) (2007), 1855-1866.

[15] M. M. Hasaballah, H. Basit, M. M. Abdelwahab, S. Bashir, A. Sanaullah and M. A. Abdelkawy, A novel Poisson-Weibull model for stress-strength reliability analysis in industrial systems: Bayesian and classical approaches, Axioms 14(9) (2024), 653.

[16] M. Jabbari Nooghabi, Shrinkage estimation, J. Stat. Comput. Simul. 86(16) (2016), 3263-3275.

[17] N. S. Kambo, A. S. Bains and H. C. Taneja, Shrinkage estimation in reliability, J. Statist. Plann. Inference 25(2) (1990), 145-158.

[18] H. J. Kim, Estimation of stress-strength reliability for the generalized half-logistic distribution, J. Korean Statist. Soc. 45(3) (2016), 321-329.

[19] S. C. Kochar, H. Mukerjee and F. J. Samaniego, The 'total time on test' transform and stochastic orderings, J. Appl. Probab. 39(2) (2002), 273-281.

[20] A. J. Lemonte and G. M. Cordeiro, The McDonald Lomax distribution: theory and applications, Journal of Data Science 11(1) (2013), 1-20.

[21] X. Li, N. Balakrishnan and M. H. Ling, Estimation under progressive censoring, J. Statist. Plann. Inference 137(2) (2007), 593-606.

[22] K. S. Lomax, Business failures: another example of the analysis of failure data, J. Amer. Statist. Assoc. 49(268) (1954), 847-852.

[23] J. S. Mehta and R. Srinivasan, Estimation of $P(Y < X)$ in the stress-strength model, J. Amer. Statist. Assoc. 66 (1971), 937-940.

[24] N. U. Nair, P. G. Sankaran and K. Jayakumar, Reliability modeling and analysis with the total time on test transform of order n, Statist. Papers 49(2) (2008), 245-262.

[25] L. Nie and W. Gui, Reliability estimation for stress-strength models based on the unit-half normal distribution, Symmetry 11(4) (2019), 837.

[26] H. M. Okasha, Estimation of stress-strength reliability for the inverse Weibull distribution, Journal of Statistics Applications and Probability 3(2) (2014), 219-229.

[27] M. Panahi Asadi, Estimation for Lomax distribution under censoring, J. Statist. Res. 45(2) (2011), 123-138.

[28] M. Panahi Asadi and M. Panahi, Estimation of stress-strength reliability for Lomax distribution, J. Stat. Comput. Simul. 81(12) (2011), 1711-1723.

[29] M. Pandey and S. K. Upadhyay, Bayesian estimation of reliability in stress-strength models, Comm. Statist. Theory Methods 14(12) (1985), 2799-2813.

[30] H. Pham and N. Turkkan, New applications of the Lorenz curve in reliability analysis, Microelectronics Reliability 34(8) (1994), 1261-1270.

[31] M. Shaked and J. G. Shanthikumar, Stochastic Orders, Springer, 2007.

[32] R. Singh and K. Anil, Stress-strength reliability model under discrete lifetime distributions, J. Indian Statist. Assoc. 45(2) (2007), 123-134.

[33] A. A. Soliman, A. H. Abd Ellah and M. Abd-Elhameed, Estimation of stress-strength reliability for the Burr XII distribution under accelerated life test model, Applied Mathematics and Information Sciences 9(2) (2015), 637-646.

[34] W. A. Thompson, Stress-strength models for reliability, J. Amer. Statist. Assoc. 63(321) (1968), 160-178.

[35] A. S. Yadav, S. K. Singh and U. Singh, Estimation of stress-strength reliability for the inverse Lindley distribution, Journal of Statistics Applications and Probability 8(1) (2019), 1-10.

[36] H. A. Yousef, Estimation of stress-strength reliability under exponential distribution, Microelectronics and Reliability 26(6) (1986), 1039-1044.

[37] H. A. Yousef, Estimation of stress-strength reliability under Weibull distribution, Microelectronics and Reliability 31(4) (1991), 743-746.

[38] H. Zakerzadeh, S. Nadarajah and M. Z. Raqab, Stress-strength reliability of the generalized half-normal distribution, Comm. Statist. Theory Methods 45(21) (2016), 6367-6381.