Far East Journal of Mathematical Sciences (FJMS)
© 2026 Pushpa Publishing House, Prayagraj, India
https://pphmjopenaccess.com
https://doi.org/10.17654/0972087126032 **P-ISSN: 0972-0871**
Volume 143, Number 2, February 2026, Pages 549-561 **E-ISSN: 2584-1246**

POTENTIAL THEORY OF BIHARMONIC FUNCTIONS ON RIEMANN SURFACES

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Abstract

This note develops a potential theory of biharmonic functions defined on parabolic or hyperbolic Riemann surfaces. The novel feature is that the proofs do not make explicit use of derivatives of functions on the surfaces. A classification theory of Riemann surfaces based on the properties of biharmonic functions and bipotentials is introduced.

Received: October 24, 2025; Accepted: December 18, 2025

2020 Mathematics Subject Classification: 31A05, 31A10, 31A30.

Keywords and phrases: biharmonic functions and bipotentials on Riemann surfaces, biharmonic Green functions.

Communicated by K. K. Azad

How to cite this article: Ibtesam Bajunaid, Potential theory of biharmonic functions on Riemann surfaces, Far East Journal of Mathematical Sciences (FJMS) 143(2) (2026), 549-561. https://doi.org/10.17654/0972087126032

1. Introduction

The theory of harmonic and subharmonic functions, logarithmic and Green potentials and the classification theory based on their properties on Riemann surfaces is a subject well-developed by mathematicians from Japan, Finland, Romania, etc. Related to this is the study of biharmonic functions on Euclidean domains, useful in the theory of elasticity and bending plates. In this note, we develop a theory of biharmonic functions on Riemann surfaces depending much on the Laplace operator as a differential operator. For this, we make use of a result of Brelot's proved years ago.

In fact, Brelot [3] has proved that if $f(x) \geq 0$ is a locally Lebesgue integrable function on the Euclidean space $\mathbb{R}^n$, $n \geq 2$, then there exists a superharmonic function $s(x)$ on $\mathbb{R}^n$ such that $-\Delta s(x) = f(x)$. Here the operator Δ is taken in the sense of distributions; the existence of such a superharmonic function is assured even if $f(x)$ is replaced by a Radon measure μ. To prove this, he makes use of the Runge-type approximation theorem for harmonic functions: If K is a compact set contained in an open set ω in $\mathbb{R}^n$ and if $h(x)$ is a harmonic function on ω and $\varepsilon > 0$ given, then there exists a harmonic function $H(x)$ on $\mathbb{R}^n$ such that $|H(x) - h(x)| < \varepsilon$ if $x \in K$.

Now such an approximation theorem is available on Riemannian surfaces also (Pfluger [8]). Using this harmonic approximation theorem and following the method of Brelot's on Euclidean spaces, we can solve the equation $\Delta u(x) = f(x)$ (respectively a Radon measure μ) on a Riemann surface R.

2. Biharmonic Potentials

Notation 1. To say that $Lu(x) = f(x)$ means that a locally integrable function u on a Riemann surface R is generated by a locally integrable function or Radon measure f on R. This generation is defined by the distributional equation $-\Delta u(x) = f(x)$, which holds at every point x on R when using a local coordinate system.

A lower semi-continuous function s, where $-\infty < s(x) \leq \infty$, is defined as superharmonic at a point z on a Riemann surface R if it exists within a neighborhood of z and satisfies the condition $Ls(z) \geq 0$. We define s as superharmonic on the entire surface R if this condition holds true for every point z on R.

To fully grasp the foundational concepts of this work, an understanding of the properties of superharmonic and harmonic functions on Riemann surfaces is essential. These include the Riemann boundary value problem, the Poisson equation, the existence of superharmonic functions with point harmonic singularities, and the classification of Riemann surfaces as either hyperbolic or parabolic. Also crucial is the Domination Principle, which states that for a locally bounded potential p harmonic outside a set A, if a positive superharmonic function s satisfies $s(x) \geq p(x)$ on A, then the inequality holds true for all points x on R. Furthermore, the representation of positive superharmonic functions as integrals, utilizing Green kernels and minimal harmonic functions, is a core concept. For a deeper dive into these topics, see Ahlfors and Sario [1], Forster [5] and Sario and Nakai [12].

Definition 1. For a locally integrable function $u(x)$ on R, let $Lu(x) = v(x)$. In this sitting, we say:

(1) u is bisuperharmonic (biharmonic) if v is superharmonic (harmonic).

(2) u is a bipotential if u and v are potentials.

(3) u is a biharmonic potential if u is a potential and v is harmonic.

A very important study of polyharmonic functions (in particular, biharmonic functions) in $\mathbb{R}^n$ is presented in Nicolesco [7] starting with the Almansi representation. We find there Liouville-Picard-Hadamard theorem for polyharmonic functions; Hadamard-Montel theorems for families of polyharmonic functions; generalized Green formula, Riquier problem etc.

Notation 2. When u_1 and u_2 are two locally integrable functions, $Lu_1 = v_1$ and $Lu_2 = v_2$, write $u_1 \succcurlyeq u_2$ if and only if $u_1 \geq u_2$ and $v_1 \geq v_2$.

Theorem 1. *If* $u \succcurlyeq 0$ *is bisuperharmonic on* R, *then it is the unique sum of a bipotential, a biharmonic potential and a non-negative harmonic function.*

Proof. Let $Lu(x) = v(x)$. Since $u \succcurlyeq 0$, v is a non-negative superharmonic function, hence v is the unique sum of a potential p_1 and a non-negative harmonic function h_1. Now there exist superharmonic functions Q_1 and s_1 on R such that $LQ_1(x) = p_1(x)$ and $Ls_1(x) = h_1(x)$.

Consequently, $Lu = p_1 + h_1 = L(Q_1 + s_1)$, so that $u = Q_1 + s_1 +$ (a harmonic function h_2). Since $u \geq 0$, we see that Q_1 has a subharmonic minorant in R so that $Q_1 =$ a potential $p_2 +$ a harmonic function on R. Similarly $s_2 =$ a potential $p_3 +$ a harmonic function on R. As a result, $u = p_2 + p_3 +$ (a harmonic function H). Since $u \geq 0$, $-H \leq p_2 + p_3$ so that $-H \leq 0$. Note that since $Lp_2 = LQ_1 = p_1$, p_2 is a bipotential; and since $Lp_3 = Ls_1 = h_1$, p_3 is a biharmonic potential.

The uniqueness of decomposition of $u \succcurlyeq 0$ as the sum of a bipotential, a biharmonic potential and a non-negative harmonic function is a consequence of the uniqueness of decomposition of a positive superharmonic function as the sum of a potential and a non-negative harmonic function. □

Theorem 2. *Let* $u(x)$ *be a non-negative bisuperharmonic function on a Riemann surface* R. *Then* $u(x)$ *has the unique integral representation*

$$u(x) = \int G_y(x)v(y)dy + \int G_y(x)h(y)dy + \int f(x)d\mu(f),$$

where $v(x)$ *is a potential,* $h(x)$ *is a non-negative harmonic function on* R, *and* μ *is a uniquely determined Radon measure on the set* Λ_1 *of minimal* *harmonic functions f on R. This is a bisuperharmonic analogue of the Riesz representation theorem.*

Remark 1. In $\mathbb{R}^n$ there are no positive biharmonic potentials for any $n \geq 2$, but there exist positive bipotentials if $n \geq 5$.

Definition 2. For a superharmonic function s on a set A,

$$R_s^A = \inf\{u,\ u \geq 0 \text{ superharmonic function on } R, \text{ and } u \geq s \text{ on } A\}.$$

Proposition 1. *If there exists a (non-harmonic) biharmonic potential $u \succcurlyeq 0$, then there exist non-zero bipotentials v on R such that $u \succcurlyeq v$.*

Proof. Let ω be a relatively compact domain in R. Then $s = Lu$ is a positive superharmonic function and $v_1 = R_s^{\omega}$ is a potential on R. Let $Lv_2 = v_1 > 0$, hence v_2 is a superharmonic function on R. Let $Lv_3 = s - v_1 \geq 0$; then v_3 also is superharmonic function.

Then $L(v_2 + v_3) = Lu$ so that $u = v_2 + v_3 +$ (a harmonic function f on R). Since $u > 0$, v_2 has a subharmonic minorant on R, hence it is the sum of a potential p_1 and a harmonic function; similarly v_3 also is the sum of a potential p_2 and a harmonic function. Thus $u = p_1 + p_2 +$ (a harmonic function g on R). Now by using the uniqueness of decomposition, $p_1 + p_2$ is the potential part of u and g is the harmonic part of u, hence $g \geq 0$. Consequently $p_1 \leq u$ and $Lp_1 = Lv_2 = v_1$. Note that p_1 is a bipotential and $u \succcurlyeq p_1$. Write $v = p_1$. ☐

Let us denote by β the class of all biharmonic potentials on R.

Theorem 3. *If $b, b_1 \in \beta$ are such that $b \succcurlyeq b_1$, then there is $b_2 \in \beta$ such that $b = b_1 + b_2$.*

Proof. Let $Lb = h$ and $Lb_1 = h_1$. Since $b \succcurlyeq b_1$, $b \geq b_1$ and $h \geq h_1$. Let $h_2 = h - h_1$ and $Ls = h_2 = Lb - Lb_1$. Hence $s + b_1 = b$. Since b and

b_1 are potentials and s is superharmonic, it can be seen that s is the sum of a potential b_2 and a nonnegative harmonic function H. Thus $Lb_2 = Ls = h_2$ and $(b_2 + H) + b_1 = b.$ Consequently by the uniqueness of decomposition $H = 0.$ Thus $b_2 \in \beta$ and $b_2 + b_1 = b.$ $\square$

The above proof contains also the proof of the following:

Proposition 2. *Suppose* b_1 *and* b_2 *are two biharmonic potentials generated by* h_1 *and* h_2, *respectively. If* $h_1 \leq h_2$, *then* $b_1 \preceq b_2$.

Theorem 4. *If* $b_1, b_2 \in \beta$, *then there exists* $b = b_1 \curlyvee b_2 \in \beta$ *with the properties*: $b \succeq b_1$ *and* $b \succeq b_2$; *and for some* $b_3 \in \beta$, *if* $b_3 \succeq b_1$ *and* $b_3 \succeq b_2$, *then* $b_3 \succeq b$.

Proof. Let $Lb_1 = h_1$ and $Lb_2 = h_2$. Then $u = \sup(h_1, h_2)$ is subharmonic and if $h_3 = h_1 \curlyvee h_2$ is the least harmonic majorant of u, then choose b in β for which $Lb = h_3$. Then $b \succeq b_1$ and $b \succeq b_2$.

If $b_3 \in \beta$ is such that $b_3 \succeq b_1$ and $b_3 \succeq b_2$, then $h_4 \geq h_1 \curlyvee h_2$, where $h_4 = Lb_3$, hence by Proposition 2, $b_3 \geq b_1 \curlyvee b_2$. $\square$

Similarly, we also have

Theorem 5. *Suppose* b_1 *and* b_2 *are two biharmonic potentials on* R. *Then there exists a biharmonic potential* $B = b_1 \curlywedge b_2$ *such that* $B \preceq b_1$ *and* $B \preceq b_2$; *and if* B' *is another biharmonic potential such that* $B' \preceq b_1$ *and* $B' \preceq b_2$, *then* $B' \preceq B$.

Minimal biharmonic potentials in β:

A biharmonic potential $b \in \beta$ is said to be a minimal biharmonic potential if $v \in \beta$ and $v \preccurlyeq b$, then $v = \alpha b$ for some $0 \leq \alpha \leq 1$. Recall that minimal harmonic functions are important in the context of the integral representation of positive harmonic functions on R.

Theorem 6. *The biharmonic potential* b *generated by* h, *is minimal if and only if the harmonic function* h *is minimal.*

Proof. Let b be a biharmonic potential, and $Lb = h$.

(1) Suppose h is minimal. Now if b_1 is a biharmonic potential, $Lb_1 = h_1$ and $b_1 \preceq b$, then $h_1 \leq h$ so that $h_1 = \alpha h$. That is $Lb_1 = \alpha Lb$. This means that $b_1 - \alpha b$ is harmonic. Since b_1 and αb are potentials, we conclude that $b_1 = \alpha b$. Hence b is a minimal biharmonic potential.

(2) On the other hand, suppose b is a minimal biharmonic potential. Take a harmonic function h_1, $0 \leq h_1 \leq h$. Suppose $b_1 \in \beta$ and $Lb_1 = h_1$. Then by Proposition 2, $b_1 \preceq b$ so that $b_1 = \alpha b$ which implies that $h_1 = \alpha h$. Hence h is a minimal harmonic function. ☐

3. Greatest Biharmonic Minorant

Theorem 7. *Suppose* u *is bisuperharmonic*, v *is bisubharmonic and* $u \succeq v$. *Then there exists a biharmonic function* b, $u \succeq b \succeq v$. *This biharmonic function* b *can be chosen such that if* b' *is another biharmonic function such that* $u \succeq b' \succeq v$, *then* $b \succeq b'$.

Proof. Let h be the greatest harmonic minorant of Lu. Then $Lu \geq h \geq Lv$, since Lv is a subharmonic function dominated by the superharmonic function Lu.

Let $LB = h$; $Lf = Lu - h$ and $Lg = Lv - h$. Then f is superharmonic and g is subharmonic. Since $Lu = Lf + LB$, $u = f + B +$ (a harmonic function on R) $= f' + B$, where f' is a superharmonic function on R. Similarly, $v = g' + B$, where g' is subharmonic on R.

Since $u \geq v$, $f' \geq g'$. Let h' be the greatest harmonic minorant of f'. Let $b = h' + B$. Then $Lb = LB = h \leq Lu$.

Moreover, $u = f' + B \geq h' + B = b$. Consequently, the biharmonic function $b \preccurlyeq u$. Similarly we show that $b \succcurlyeq v$. Thus $u \succcurlyeq b \succcurlyeq v$.

Suppose b' is another biharmonic function such that $u \succcurlyeq b'$. Then $u \geq b'$ and $Lu \geq Lb'$. Here Lu is superharmonic and Lb' is harmonic. Since h is the greatest harmonic minorant of Lu, we conclude that $Lu \geq h \geq Lb'$. Since, $h = LB = Lb$, $Lb \geq Lb'$ so that $b' = b +$ a subharmonic function w.

Now $f' + B = u \geq b' = b + w = (h' + B) + w$.

Since B is real valued except on a polar set E, $f' \geq h' + w$ outside E. Since f' is superharmonic and $h' + w$ is subharmonic, $f' \geq h' + w$ everywhere on R. But h' is the greatest harmonic minorant of f', so that $w \leq 0$. Consequently, $b' = b + w \leq b$; since $Lb' \leq Lb$ also, $b \succcurlyeq b'$. ☐

Definition 3. The biharmonic function b in the above theorem is termed the greatest biharmonic minorant of u.

Theorem 8. *A bisuperharmonic function* $u \succcurlyeq 0$ *is a bipotential if and only if the greatest biharmonic minorant of u is* 0.

Proof. Suppose the greatest biharmonic minorant of u is 0. Then Lu should be a potential; otherwise take a harmonic function h, $0 < h < Lu$. Let $Lb = h$. Then b is a biharmonic function such that $0 \preccurlyeq b \preccurlyeq u$, by Proposition 2, a contradiction. Hence $h = 0$. This implies that Lu is a potential. Consequently, $u \geq 0$ is a superharmonic function. Actually, u is a potential. For suppose H is the greatest harmonic minorant of u. Then H can be considered a biharmonic function so that $0 \preccurlyeq H \preccurlyeq u$. By assumption then $H = 0$. Thus u and Lu are potentials on R, that is, u is a bipotential.

Conversely, suppose u is a bipotential on R. Let B be a biharmonic function such that $0 \preccurlyeq B \preccurlyeq u$. Since $LB \geq 0$ is harmonic and Lu is a potential, then $LB = 0$; that is B is harmonic. Since u is a potential and

$0 \leq B \leq u,$ then $B = 0.$ This shows that the greatest biharmonic minorant of u is 0. □

4. Biharmonic Extension

In a hyperbolic Riemann surface R, if h is a harmonic function near infinity (that is, defined outside a compact set) in R, then it is of the form: $h(x) = H(x) + f(x)$ near infinity where $|f(x)| \leq p(x)$ for a potential p in R and H is a uniquely determined harmonic function on R. Here it is convenient to consider h as a local integrable function on R that is harmonic outside a compact set in R. (See, Rodin and Sario [9], Anandam [2]).

Theorem 9 (Biharmonic extension)**.** *Let u be a biharmonic function near infinity in a hyperbolic Riemann surface R. Then u is of the form*: $u(x) = B(x) + b(x)$ *near infinity, where B is a biharmonic function on R and* $|Lb(x)| \leq p(x)$ *for a potential p on R. If* $u(x) = B'(x) + b'(x)$ *is another such representation*, *then for a harmonic function v on R,* $B'(x) = B(x) + v(x)$ *and* $b'(x) = b(x) - v(x).$

Proof. Considering u as a locally integrable function on R that is biharmonic outside a compact set K, the function $h(x) = Lu(x)$ is harmonic near infinity. Then $h(x) = H(x) + h_1(x),$ where $H(x)$ is harmonic on R and $|h_1(x)| \leq p(x),$ where p is a potential on R. Let $LB_1(x) = H(x)$ so that B_1 is biharmonic on R. Let $Lb_1(x) = h_1(x)$ so that b_1 is biharmonic near infinity.

Now $Lu(x) = h(x) = H(x) + h_1(x) = LB_1(x) + Lb_1(x)$ which leads to $u(x) = B_1(x) + b_1(x) + h_2(x)$ outside a compact set. Write $h_2(x) = H_1(x) + h_3(x),$ where H_1 is harmonic on R and $|h_3(x)| \leq p_3(x)$ near infinity for a potential p_3 on R. Hence, outside a compact set in R,

$$u(x) = B_1(x) + b_1(x) + [H_1(x) + h_3(x)]$$

$$= [B_1(x) + H_1(x)] + [b_1(x) + h_3(x)]$$
$$= B(x) + b(x).$$

Here, $Lb(x) = Lb_1(x) + Lh_3(x) = Lb_1(x) = h_1(x)$ outside a compact set; hence $|Lb(x)| \leq p(x)$.

To prove the uniqueness of the representation up to a harmonic function, suppose $u(x) = B'(x) + b'(x)$, and $|Lb'(x)| \leq p'(x)$. Then $B(x) - B'(x)$ is biharmonic on R, and $|L(B - B')| = |L(b' - b)| \leq p' + p$. Hence the harmonic function $L(B - B') = 0$. That is, $B - B' = v$ is a harmonic function on R and the theorem follows. ☐

5. Biharmonic Green Function

Kurata and Yamasaki [6], in the framework of discrete harmonic functions on an infinite network and on homogeneous infinite trees, study the properties of multiharmonic Green functions. Earlier, Yamasaki [11] had published an article on biharmonic Green function of an infinite network.

In this section, we discuss the existence and the properties of biharmonic Green functions on hyperbolic Riemann surfaces.

In a hyperbolic Riemann surface R, let G_y be the Green potential having pole at y. Write $Ls(x) = G_y(x)$ so that s is a superharmonic function that is bisuperharmonic on R and biharmonic outside y. But it is not necessary that s is a potential on R, even up to an additive harmonic function. In this context, we have the following theorem:

Theorem 10. *The solution s of the equation $Ls(x) = G_y(x)$ is a potential up to an additive harmonic function if and only if positive bipotentials exist on R.*

Lemma 1. *If u is a superharmonic function near infinity on a hyperbolic Riemann surface R, then u is of the form $u(x) = v(x) + h(x)$, where v is superharmonic on R and h is a bounded harmonic function near infinity.*

Proof of Theorem 10. If s is the sum of a potential w and a harmonic function on R, then $Lw(x) = Ls(x) = G_y(x)$; then w is actually a bipotential on R. On the other hand, suppose there is a positive bipotential q on R generated by a potential p, $Lq(x) = p(x)$, and $Ls(x) = G_y(x)$. Let Σ be a domain in R that is the pre-image of a disc in $\mathbb{R}^2$, containing y. Let $D[G_y(x)]$ be the Dirichlet solution on $R \backslash \Sigma$ with boundary values G_y on $\partial\Sigma$ and 0 at the point at infinity. Then $D[G_y(x)] = G_y(x)$ on $R \backslash \Sigma$. Note that for some $\alpha > 0$, $\alpha p(x) \geq G_y(x)$ on $\partial\Sigma$ and consequently, $\alpha p(x) \geq G_y(x)$ on $R \backslash \Sigma$. That is, $L[\alpha q(x)] \geq Ls(x)$ hence $\alpha q(x) = s(x) + u(x)$ near infinity except on a polar set, where u is a superharmonic function near infinity. Consequently since all the functions are superharmonic, $\alpha q(x) = s(x) + u(x)$ for all x in a neighbourhood of the point at infinity. Then by Lemma 1, $\alpha q(x) = s(x) + v(x) + h(x)$ is near infinity. This implies, since $q(x) > 0$, that s has a subharmonic minorant outside a compact set in R, consequently, s is the sum of a potential w and a harmonic function on R.

$\square$

Remark 2. From the above theorem, we conclude that if there are bipotentials on R, then there is a bipotential Q_y on R such that $LQ_y(x) = G_y(x)$ for all x in R. We refer to the unique bipotential Q_y as the biharmonic Green potential on R having pole at y.

Definition 4. A superharmonic function on R is said to be *admissible* if it has a harmonic minorant outside a compact set.

Proposition 3. *In a hyperbolic Riemann surface R without the biharmonic Green kernel, let $s(x) > 0$ be a superharmonic function on R. If $Lu(x) = s(x)$, then the superharmonic function $u(x)$ is not admissible.*

Proof. Suppose on the contrary that u is admissible, that is u has a harmonic minorant outside a compact set on R; then u has a harmonic minorant on the whole of R [2]. Consequently, u is the sum of a positive potential v and a harmonic function on R so that $Lv(x) = s(x)$. Now write s as the sum of a potential p and a non-negative harmonic function h. Write $Lq(x) = p(x)$ and $LH(x) = h(x)$ so that $Lv(x) = s(x) = L[q(x) + H(x)]$. Hence $v(x) = q(x) + H(x) + w(x)$, where q and H are superharmonic and w is harmonic on R. Consequently, since $v(x) > 0$, $q(x)$ has a subharmonic minorant on R so that $q(x)$ is the sum of a potential $q_1(x)$ and a harmonic function on R. Then $Lq_1(x) = Lq(x) = p(x)$ so that q_1 is a positive bipotential on R, so that the biharmonic Green kernel exists on R (Theorem 10). This contradicts the hypothesis that the biharmonic Green kernel cannot be defined on R, thus proving the proposition. □

6. Integral Representation of Bipotentials

Let u be a bipotential generated by a potential v and $Lv = \mu$ is a Radon measure. Then,

$$\begin{aligned} u(x) &= \int v(z) G_z(x) dz \\ &= \int \left[\int G_y(z) d\mu(y) \right] G_z(x) dz \\ &= \int \left[\int G_y(z) G_z(x) dz \right] d\mu(y) \\ &= \int Q_y(x) d\mu(y). \end{aligned}$$

Note that since u and v are potentials, the integrals are convergent and changing the order of integration is justified.

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