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TRANSMISSION DYNAMICS OF CORONAVIRUS DISEASE DURING THE SECOND WAVE: A MATHEMATICAL STUDY IN THE HOLY CITY UJJAIN

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Abstract

In this paper, we attempt to study the dynamics of coronavirus disease during the second wave in the Indian district of Ujjain. For this, we use a classical epidemic model of susceptible, exposed, infected and recovered population with saturated incidence and treatment rates. We derive an associated dynamically consistent nonstandard finite difference scheme and fit it to the corresponding cumulative number of infected cases reported by the government of Madhya Pradesh. By

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using the method of least square, we determine the basic reproduction number and other model parameters. In order to determine the relative importance of model parameters, we perform sensitivity analysis. The current study shows that the disease prevalence was highest in early May, and the basic reproduction number was 4.275 at that time. Along with this, the study highlights the need to follow the COVID-19 protocol and confirms that the disease spreads earlier in large cities than in smaller cities.

1. Introduction

Coronavirus is a highly contagious disease caused by the SARS-COV-2 virus. The first case of the coronavirus was reported in the Wuhan province, China in December 2019 and WHO declared it a global pandemic just two months later in March 2020 [31]. General symptoms of the disease are fever, cough and tiredness. However, symptoms, severity and treatment strategy were changed with the emergence of new variants and their mutations [14]. As in case of India, who witnessed three waves of disease dissemination and experienced different situations every time. The first case of coronavirus in India was recognized in January 2020 in Kerala [25]. The first wave occurred due to SARS-COV-2 virus while its variants brought the second and third. Transmission efficiency of the virus increased in the second wave whereas the incubation period reduced. In the second wave, the whole country was facing an acute shortage of hospital beds, oxygen supply, medicines and ventilators that was not common in the first wave [16]. By the time of the second wave, a lot of research had been done and doctors were aware of the line of treatment. On the other hand, mathematicians were trying to study the spread of the disease using the mathematical model.

Mathematical models are being used to study the nature of the infectious diseases since long time. Some of the early studies set milestones in the study of infectious diseases modelling; they proved that the dynamics of the disease depends on the populations and transmission rates [18, 27]. Taking this further, mathematicians divided the basic SIR, SIS, and SEIR models into compartments based on ecology, demography, migration, traits, sex,

vaccination, treatment, and many more to provide an in-depth analysis of disease spread [7, 17]. Mathematical models are established mainly on the deterministic and stochastic approaches. Even among them, discrete models are accurate, preserve dynamical properties of continuous models and, describe the infectious disease easily [20]. To control and predict the spread of any disease, it is necessary to have adequate knowledge about its physical characteristics. Hence, parameter estimation plays an important role in the infectious disease modelling that is performed by using some robust numerical methods and method of least square [4, 6, 23]. However, NSFD scheme is better than the standard numerical methods like Euler and Runge Kutta method as these methods generate oscillations and unsteady states when step size decreases at critical level [32].

Since the emergence of the coronavirus, many researchers from different countries including India have investigated the nature of the virus and its impact. This includes prediction of disease parameters through clinical and mathematical studies. Previous studies suggest that control strategies such as lockdown, social distancing, and hospitalization of infected individuals and isolation of asymptomatic infected individuals can reduce the disease outbreak and peak prevalence [6, 26, 31]. Incubation period, infectious period, basic reproduction number are some of the important characteristics that best describe any infectious disease and help in designing control strategies. Of which, the basic reproduction number is the most sensitive to the transmission rate from susceptible to infected population [25]. According to the Centers for Disease Control and Prevention, the incubation period of SARS-COV-2 ranges from 2 to 14 days with median 5.2 days whereas the infectious period from 10 to 20 days [26]. A study for estimating the recovery period in India indicates that average recovery period in the state Madhya Pradesh during the second wave was 50.348 to 92.814 days while it was 5 to 68 days for India [10]. Most of the symptomatic individuals develop symptoms between 8-16 days of infection [19]. In the present work, we study the transmission dynamics of coronavirus in district Ujjain, India. The first COVID-19 death in the state Madhya Pradesh was reported in Ujjain district on 25/03/2020 whereas the total reported COVID-19 deaths in the country

were only 10 [28]. Among the three waves, the second was horrific and Ujjain was among the top five most affected districts of the state. Total reported cases in Ujjain were 5327 and 7099 on 15 March 2021 and 10 April, respectively, that increased to 17108 on 15 May 2021 [28]. For a small district like Ujjain whose population is nearly twenty lakhs, this increase in the cases was worrisome. The remaining paper is as follows: In Section 2, we discuss the dynamical properties of the deterministic model and the associated NSFD scheme. In Section 3, we perform parameter estimation by fitting NSFD scheme. We perform sensitivity analysis in Section 4. In Section 5, we discuss mathematical results by linking them to the actual situation. Finally, we present some conclusions in Section 6.

2. Materials and Methods

2.1. Data

We used the data of cumulative number of infected cases of COVID-19 of the district Ujjain. We considered a longer span of six months so that we can analyze the disease dynamics before and after the peak as well. Data is included from 01/01/2021, which is long before the peak. Since the daily new cases became zero at the end of June and only three new cases were reported in the month of July, only the data up to 30/06/2021 is included. The data is available in the repository of the Government of Madhya Pradesh [28]. Figures 1 and 2 depict the daily reported cases of new infection and the cumulative number of infected cases.

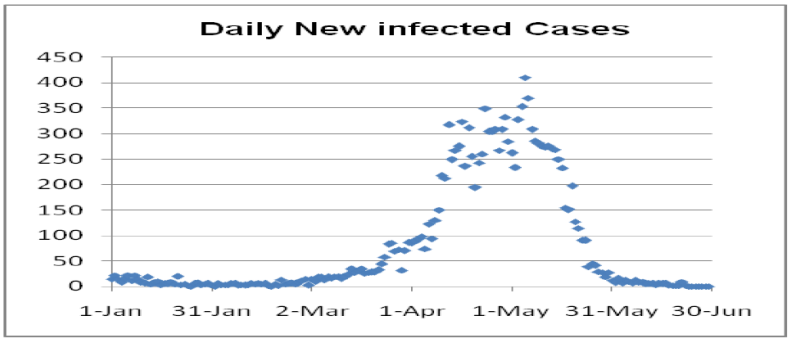

Figure 1. Daily reported number new infected cases.

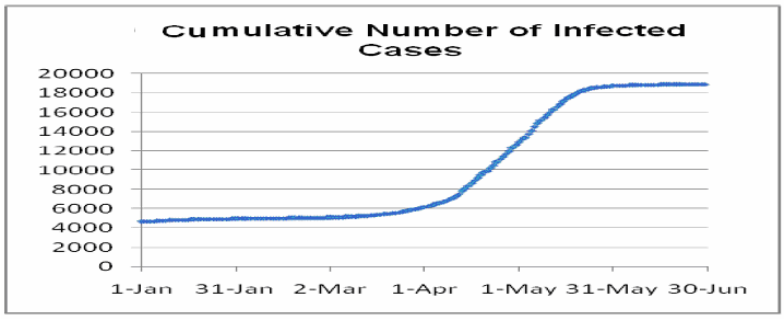

Figure 2. Cumulative number new infected cases.

2.2. Mathematical model (deterministic model)

The present model is based on the SEIRS model discussed by Oluyori et al. [24] with minor changes. They included the disease-induced death in the recovered class but we did not use it in our study. We considered four classes of individuals, which are as follows: susceptible individuals (S), exposed individuals (E), infected individuals (I) and recovered individuals (R). υ is the constant recruitment rate in the susceptible population. μ is the natural death rate. β is the rate of transmission, α is the saturation parameter to measure the inhibitory effect, ω is the rate of re-infection, σ is the rate of conversion in exposed to infected, μ_1 is the disease induced death rate, γ is the natural recovery rate of an infected individual, $\frac{aI}{1+bI}$ is the saturated treatment response. Mathematical model used for the disease transmission is expressed as follows:

$$\begin{aligned}
\frac{dS}{dt} &= \upsilon + \omega R - \frac{\beta SI}{1+\alpha I} - \mu S, \\
\frac{dE}{dt} &= \frac{\beta SI}{1+\alpha I} - \mu E - \sigma E, \\
\frac{dI}{dt} &= \sigma E - \mu I - \mu_1 I - \gamma I - \frac{aI}{1+bI}, \\
\frac{dR}{dt} &= \frac{aI}{1+bI} + \gamma I - \omega R - \mu R.
\end{aligned} \tag{1}$$

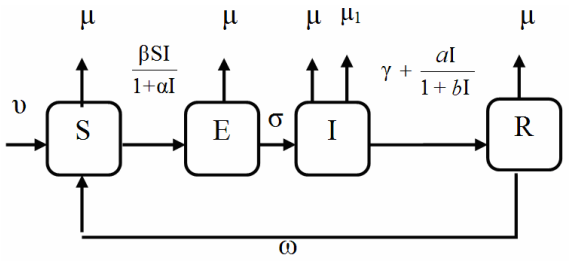

Figure 3. Transmission diagram of the coronavirus.

2.2.1. Positivity of the solutions

Lemma 1. *If all the initial values and the parameter values are positive, then the solutions* $S(t)$, $E(t)$, $I(t)$ *and* $R(t)$ *are positive for all* $t \geq 0$.

Proof. Consider the first equation of model (1):

$$\frac{dS}{dt} = \upsilon + \omega R - \frac{\beta SI}{1+\alpha I} - \mu S$$

$$\Rightarrow \frac{dS}{dt} \geq -\frac{\beta SI}{1+\alpha I} - \mu S$$

$$\Rightarrow \frac{dS}{S} \geq -\left(\frac{\beta I}{1+\alpha I} + \mu\right)dt.$$

After solving, we get

$$S(t) \geq e^{-\left(\frac{\beta I}{1+\alpha I}+\mu\right)t} S(0)$$

$$\Rightarrow S(t) \geq 0.$$

Similarly, we can prove the positivity of $E(t)$ and $R(t)$. Positivity of $I(t)$ is proved in the later section.

2.2.2. Boundedness of the solution

Lemma 2. *The continuous model* (1) *defines the dynamical system on*

$$\Omega = \left\{(S(t),\, E(t),\, I(t),\, R(t)) \in \Re_+^4 : 0 \leq (S(t) + E(t) + I(t) + R(t)) \leq \frac{\upsilon}{\mu}\right\}.$$

Proof. Consider the population $N(t)$ at time t, where

$$N(t) = S(t) + E(t) + I(t) + R(t).$$

Adding all the equations of system (1), conservation law $\frac{dN(t)}{dt} \leq \upsilon - \mu N(t)$ can be proved.

After solving and evaluating, we get $N(t) = \frac{\upsilon}{\mu}$, as $t \to \infty$. This shows that $0 \leq N(t) \leq \frac{\upsilon}{\mu}$.

2.2.3. Disease free equilibrium (DFE) point

Disease free equilibrium state for (1) is as follows:

$$S^0 = \frac{\upsilon}{\mu}, \quad E^0 = 0, \quad I^0 = 0, \quad R^0 = 0.$$

2.2.4. Basic reproduction number

Next generation matrix determined [11] is

$$FV^{-1} = \begin{bmatrix} \frac{\beta\sigma S^0}{(\mu+\sigma)(\mu+\mu_1+\gamma+a)} & \frac{\beta S^0}{(\mu+\mu_1+\gamma+a)} \\ 0 & 0 \end{bmatrix}.$$

Therefore,

$$R_0 = \frac{\beta\sigma\upsilon}{\mu(\mu+\sigma)(\mu+\mu_1+\gamma+a)}.$$

2.3. NSFD scheme

To construct a non-standard finite difference (NSFD) scheme corresponding to deterministic model (1), we used the method proposed by Mickens [21]. For times $t_k = kh$, the number of susceptible, exposed, infected and recovered individuals are represented by S_k, E_k, I_k, R_k, respectively, where h is the integration step size. We formed the following

NSFD scheme that preserves the dynamical properties of deterministic model (1):

$$\frac{S_{K+1}-S_K}{\varphi(h)} = \upsilon + \omega R_{K+1} - \frac{\beta S_{K+1} I_K}{1+\alpha I_K} - \mu S_{K+1},$$

$$\frac{E_{K+1}-E_K}{\varphi(h)} = \frac{\beta S_{K+1} I_K}{1+\alpha I_K} - \mu E_{K+1} - \sigma E_{K+1},$$

$$\frac{I_{K+1}-I_K}{\varphi(h)} = \sigma E_{K+1} - \mu I_{K+1} - \mu_1 I_{K+1} - \gamma I_{K+1} - \frac{aI_{K+1}}{1+bI_K},$$

$$\frac{R_{K+1}-R_K}{\varphi(h)} = \frac{aI_{K+1}}{1+bI_K} + \gamma I_{K+1} - \omega R_{K+1} - \mu R_{K+1}, \tag{2}$$

where the denominator function $\varphi(h) = \dfrac{e^{\mu h}-1}{\mu}$ [21], we chose $\varphi(h) = h$ [2]. Therefore, the derived NSFD scheme is as follows:

$$\frac{S_{K+1}-S_K}{h} = \upsilon + \omega R_{K+1} - \frac{\beta S_{K+1} I_K}{1+\alpha I_K} - \mu S_{K+1},$$

$$\frac{E_{K+1}-E_K}{h} = \frac{\beta S_{K+1} I_K}{1+\alpha I_K} - \mu E_{K+1} - \sigma E_{K+1},$$

$$\frac{I_{K+1}-I_K}{h} = \sigma E_{K+1} - \mu I_{K+1} - \mu_1 I_{K+1} - \gamma I_{K+1} - \frac{aI_{K+1}}{1+bI_K},$$

$$\frac{R_{K+1}-R_K}{h} = \frac{aI_{K+1}}{1+bI_K} + \gamma I_{K+1} - \omega R_{K+1} - \mu R_{K+1}. \tag{3}$$

After rearranging and simplifying, we get an explicit form

$$S_{K+1} = \left(\frac{LHF}{(DLHF - CJGP)}\right)\left(S_K + \upsilon h + \frac{CR_K}{H} + \frac{CJI_K}{LH} + \frac{CJG}{LHF} E_K\right),$$

$$E_{K+1} = \frac{E_K + PS_{K+1}}{F},\ I_{K+1} = \frac{FI_K + G(E_K + PS_{K+1})}{LF},$$

$$R_{K+1} = \frac{LFR_K + J[FI_K + G(E_K + PS_{K+1})]}{LHF}, \tag{4}$$

where

$$D = 1 + \mu h + \frac{\beta h I_K}{1 + \alpha I_K}, \quad P = \frac{\beta h I_K}{1 + \alpha I_K}, \quad C = \omega h, \quad G = \sigma h,$$

$$F = (1 + \mu h + \sigma h), \quad L = 1 + \left(\mu + \mu_1 + \gamma + \frac{a}{1 + bI_K}\right)h,$$

$$J = \left(\gamma + \frac{a}{1 + bI_K}\right)h, \quad H = (1 + \mu h + \omega h).$$

2.3.1. Positivity of the solution

Lemma 3. *If all the initial values and the parameter values are positive, then the solutions* S_{k+1}, E_{k+1}, I_{k+1} *and* R_{k+1} *are positive for all* $k \geq 0$.

Proof. E_{k+1}, I_{k+1} and R_{k+1} depend on S_{k+1}. It is sufficient to prove that $S_{k+1} \geq 0$. As all the parameter values and the initial values are positive,

$$DLHF - CJGP$$
$$= (1 + \mu h + P)[1 + (\mu + \mu_1)h + J](1 + \mu h + C)(1 + \mu h + G) - CJGP > 0$$
$$\Rightarrow S_{K+1} \geq 0.$$

2.3.2. Boundedness of the solution

Lemma 4. *The discrete model* (2) *defines the dynamical system on*

$$\Omega = \left\{(S_K, E_K, I_K, R_K) \in \Re_+^4 : 0 \leq (S_K + E_K + I_K + R_K) \leq \frac{\upsilon}{\mu}\right\}.$$

Proof. Adding all equations of NSFD scheme (3),

$$\frac{N_{K+1} - N_K}{h} \leq \upsilon - \mu N_{K+1}.$$

By applying the Gronwall inequality [32], we have the following:

If $0 < N(0) < \frac{\upsilon}{\mu}$, then $N_k \to \frac{\upsilon}{\mu}$ as $k \to \infty$. Therefore, $0 \leq N_k \leq \frac{\upsilon}{\mu}$.

2.3.3. Disease free equilibrium point and local stability

Deterministic and discrete models have same equilibrium points [1, 20, 29]. Assume that the disease free equilibrium point for the NSFD scheme (2) is $(S_N^0, E_N^0, I_N^0, R_N^0)$. Jacobian matrix J^0 of the linearization of the NSFD scheme (2) at disease free equilibrium point (S^0, E^0, I^0, R^0) of the deterministic model (1) is as follows:

$$J^0 = \begin{bmatrix} \frac{1}{(1+\mu h)} & \frac{CG(\gamma+a)h}{HF(1+\mu h)[1+(\mu+\mu_1+\gamma+a)h]} & A1 & \frac{C}{H(1+\mu h)} \\ 0 & \frac{1}{F} & \frac{\beta h \upsilon}{F\mu} & 0 \\ 0 & \frac{G}{[1+(\mu+\mu_1+\gamma+a)h]F} & \frac{(F\mu+G\beta h\upsilon)}{F\mu[1+(\mu+\mu_1+\gamma+a)h]} & 0 \\ 0 & \frac{(\gamma+a)hG}{HF[1+(\mu+\mu_1+\gamma+a)h]} & \frac{(F\mu+G\beta h\upsilon)(\gamma+a)h}{HF\mu[1+(\mu+\mu_1+\gamma+a)h]} & \frac{1}{H} \end{bmatrix},$$

where

$$A1 = \frac{\left\{\frac{Ch(1+\mu h)(\gamma+a)}{H[1+(\mu+\mu_1+\gamma+a)h]} - \left(\frac{\upsilon}{\mu}+\upsilon h\right) \cdot \beta h\left(1 - \frac{CG(\gamma+a)h}{HF[1+(\mu+\mu_1+\gamma+a)h]}\right)\right\}}{(1+\mu h)^2}.$$

NSFD scheme (2) converges to the equilibrium point if the modulus of all the eigenvalues of the Jacobian matrix J^0 is less than 1 [29]. The Jacobian matrix J^0 has 4 eigenvalues, two of which are as follows:

$$\lambda_1 = \frac{1}{H} < 1, \quad \lambda_2 = \frac{1}{(1+\mu h)} < 1.$$

For further proof, we use following lemma:

Lemma 5. *The roots of the quadratic polynomial* $\lambda^2 - A\lambda + B = 0$, *satisfy* $|\lambda_i| < 1$; $i = 1, 2$ *if and only if following conditions hold* [29]:

(i) $B < 1$,

(ii) $1 + A + B > 0$,

(iii) $1 - A + B > 0$.

Proof. The remaining eigenvalues of Jacobian matrix J^0 are given by

$$\lambda^2 - \left\{\frac{1}{F} + \frac{1}{[1+(\mu+\mu_1+\gamma+a)h]}\left(1+\frac{G\upsilon\beta h}{\mu F}\right)\right\}\lambda$$

$$+\left\{\frac{1}{F}\left[\frac{1}{[1+(\mu+\mu_1+\gamma+a)h]}\left(1+\frac{G\upsilon\beta h}{\mu F}\right)\right]\right\}$$

$$-\left(\frac{G}{[1+(\mu+\mu_1+\gamma+a)h]F}\right)\frac{\beta h\upsilon}{\mu F} = 0,$$

where

$$A = \frac{1}{F} + \frac{1}{[1+(\mu+\mu_1+\gamma+a)h]}\left(1+\frac{G\upsilon\beta h}{\mu F}\right),$$

$$B = \frac{1}{F[1+(\mu+\mu_1+\gamma+a)h]}.$$

(i) $B = \dfrac{1}{F[1+(\mu+\mu_1+\gamma+a)h]} < 1,$

(ii)

$$1 + A + B = 1 + \left\{\frac{1}{F} + \frac{1}{[1+(\mu+\mu_1+\gamma+a)h]}\left(1+\frac{G\upsilon\beta h}{\mu F}\right)\right\}$$

$$+\frac{1}{F[1+(\mu+\mu_1+\gamma+a)h]} > 0.$$

First and second conditions are obvious as all the parameters are positive. We need to prove only third condition.

(iii)

$$1 - A + B = 1 - \left\{ \frac{1}{F} + \frac{1}{[1 + (\mu + \mu_1 + \gamma + a)h]} \left(1 + \frac{G\upsilon\beta h}{\mu F} \right) \right\}$$

$$+ \frac{1}{F[1 + (\mu + \mu_1 + \gamma + a)h]}.$$

$1 - A + B > 0$ if $F[1 + (\mu + \mu_1 + \gamma + a)h] - [1 + (\mu + \mu_1 + \gamma + a)h] - F - \frac{G\upsilon\beta h}{\mu} + 1 > 0,$

which fulfilled only when $\frac{G\upsilon\beta h}{\mu(\sigma h + \mu h)(\mu + \mu_1 + \gamma + a)h} < 1$ that is $R_0 < 1$.

It can be stated that all the conditions of Lemma 5 are satisfied if $R_0 < 1$. Thus we have the following theorem:

Theorem. (i) *The deterministic model* (1) *and the associated discrete model* (2) *have the same disease free equilibrium point* (S^0, E^0, I^0, R^0).

(ii) *If* $R_0 < 1$, *then the disease free equilibrium point* (S^0, E^0, I^0, R^0) *is stable*.

(iii) *The stability of the disease free equilibrium point* (S^0, E^0, I^0, R^0) *is independent of the step size h*.

Hence, NSFD scheme (2) preserves the dynamical consistency of deterministic model (1).

3. Parameter Estimation

Infection spread was not uniform throughout the study period. So for better prediction of parameters, we divide the entire period into four phases. Phase-I (01/01/2021 to 28/02/2021): when the daily new infection was very few that is almost in single digit. Phase-II (01/03/2021 to 09/04/2021): when the daily new cases started to increase. Phase-III (10/04/2021 to

14/05/2021): when daily new cases increased rapidly and reached the peak. Phase-IV (15/05/2021 to 30/06/2021): when the daily new cases started to decrease and the infection was about to an end. Here, recruitment rate (υ) is 102.5548432 and natural death rate (μ) is 0.00001956164384. We calculate υ and μ according to district population, birth and death rates. Planning and Statistics Office, District Ujjain, provided the birth and death rates while the population of the district is available on the official website of National Informatics Centre, District Centres (Ujjain, Madhya Pradesh), Government of India [22]. Since the natural immunity persists for 8 months after COVID-19 infection, the re-infection rate (ω) has been assumed to be 0.042 [33]. The entire population of Ujjain district is considered susceptible to infection, except those who have been infected during the last 8 months preceding the first day of each phase. Initial values for infected and recovered populations are taken from the Health Bulletin [28]. Since initial values are not given for the exposed population, $E(0)$ is assumed for all four phases. Table 1 shows the initial values for the populations.

Table 1. Initial values for the susceptible, infected, recovered and exposed populations

	$S(0)$	$I(0)$	$R(0)$	$E(0)$
Phase-I	1982334	4668	4340	4700
Phase-II	1982637	5086	4902	5100
Phase-III	1981094	7099	5765	7100
Phase-IV	1972066	17108	13785	17500

Along with the initial values given in Table 1, we fit the model (4) to the real reported data and perform parameter estimation in Excel Solver by applying the method of least square as described by [8]. The method of least square is applied to minimize the summation of squared error $\sum(I_{\text{predicted}} - I_{\text{measured}})^2$ subjected to the NSFD scheme (4). $I_{\text{predicted}}$ is the numerical solution of the NSFD scheme (4) corresponding to the cumulative number of infected cases and I_{measured} is the cumulative number

of infected cases reported by the Government of Madhya Pradesh via Health Bulletin [28]. In the current study, model parameters have been estimated taking into account the incubation period and infectious period provided by the Centers for Disease Control and Prevention [26]. Table 2 represents the estimated values of parameters.

Table 2. List of estimated model parameters

Parameter	Phase-I	Phase-II	Phase-III	Phase-IV
β	3.2501E-08	5.6E-08	0.00000007	3.12E-08
σ	0.071428571	0.0999	0.145	0.075
γ	0.053	0.07	0.05	0.05
a	0.00000001	0.0012884	0.00128836	0.01
b	0.002585774	0.0929999	0.0929999	0.002586
α	9.99285E-07	9.933E-09	9.3267E-06	9.93E-09
μ_1	0.01	0.024533	0.03453296	0.011
R_0	2.703044425	3.0626898	4.27460611	2.302577

With the obtained parameter values of Table 2 and initial values of Table 1, numerical simulation of the deterministic model (1) is performed in MATLAB using ode45. Figure 4 shows the cumulative number of infected individuals and proves that the infected population remains positive for the given set of parameters if the initial value is positive.

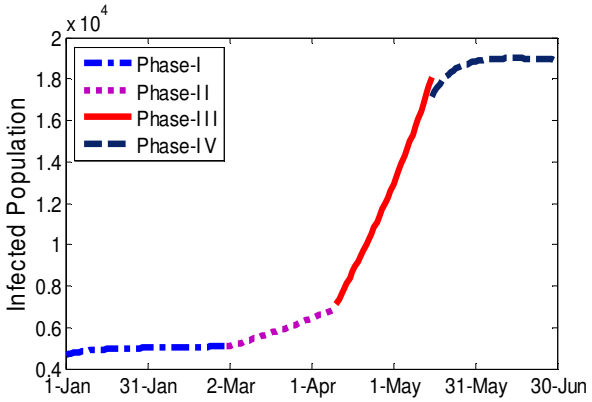

Figure 4. Time series for the cumulative number of infected by the model (1).

4. Sensitivity Analysis

"The normalized forward sensitivity index of a variable u that depends differentiably on a parameter p is defined as: $\gamma_P^u = \frac{\partial u}{\partial P} \times \frac{P}{u}$" [9]. Normalized sensitivity indices of R_0 with respect to some parameters have been calculated using baseline values given in Table 3.

Table 3. Normalized sensitivity indices of R_0 to some parameters

Parameter	Phase-I	Phase-II	Phase-III	Phase-IV
μ	–1.00060	–1.00040	–1.00040	–1.00050
β	1	1	1	1
υ	1	1	1	1
γ	–0.84100	–0.73038	–0.58248	–0.70403
μ_1	–0.15868	–0.25598	–0.40229	–0.15489
a	0.00000	–0.01344	–0.01501	–0.14081
σ	0.00028	0.00020	0.00014	0.00027

Table 3 represents the normalized sensitivity indices of R_0 with respect to parameters. It is clear that the trend for the normalized sensitivity index is the same for all phases. $\gamma_p^u = 1$ indicates that an increase (decrease) in p by $x\%$ increases (decreases) u by the same percentage [5]. The basic reproduction number is highly sensitive to the transmission rate, recruitment rate and the natural death rate. The physical meaning of $\gamma_\gamma^{R_0} = -0.84100$ is that 10% increase in γ decreases R_0 by 8.41%. Recovery rates and disease-induced mortality are also significant factors that affect disease spread, while the conversion rate of exposure to infected individuals barely affects the basic reproduction number.

5. Discussion

Phase-I. Ujjain district is spread over 6091 Sq. km, out of which only 194 Sq. km is urban, and the remaining 5897 Sq. km comes under rural areas

[13]. After the end of the first wave, in early 2021, India reported less than 15,000 COVID-19 cases per day in January and February. Similarly, Ujjain also reported daily new cases in single digits. The estimated basic reproduction number during this period was $R_0 = 2.703$ and the incubation period was 14 days. In other words, we can say that the virus was not severe and the most of the infected individuals were asymptomatic, and hence, disease induced death rate was low.

Phase-II. With the beginning of March, the COVID-19 cases in India including Ujjain started increasing but at a slow pace. However, urban areas contributed 48% to the total new confirmed cases of the country in the March that increased to 52% in the beginning of April [15]. In addition, people had a perception that COVID-19 can harm only the people living in global cities, the elderly, immunocompromised, and those with comorbidities. Therefore, despite government restrictions on mass gatherings and the mandatory requirement of masks, people were not following COVID-19 protocols like wearing masks and maintaining social distancing. Not only this, the second wave of the COVID-19 came with some new symptoms such as headache, pink eyes, diarrhoea and hearing impairment [3]. Furthermore, some of the severe symptoms like oxygen deprivation, COVID pneumonia or acute respiratory distress syndrome (ARDS), and chest pain, which were seen in the first wave in the later stages of infection, were seen in the early stages of the disease [30]. Thus, incubation period reduced to 10 days and disease induced death rate increased. Some of the patients with these symptoms had their RTPCR report negative and hence they were not recognized as COVID-19 patient. However, their CT and chest scan were showing the damage in lungs. Later, when doctors observed that the symptoms are only due to the coronavirus, they started treating them as COVID-19 patients. Along with this, after identifying the new symptoms, symptomatic patients with negative RTPCR report were also isolated, quarantined or admitted to COVID-19 hospitals. In view of such situation, the state government and local administration took some initial steps like early closure of the market and Sunday lockdown [12]. Yet, by then disease

had spread and from March end, the number of COVID-19 cases in Ujjain started increasing fast and the basic reproduction number increased to 3.063 during this phase.

Phase-III. In addition to the pre-existing conditions, towards the end of March, the COVID-19 cases started increasing rapidly in metros like Mumbai, Delhi and Pune; state governments imposed night curfews, weekend lockdowns resulting in the migration of workers to their hometowns. In addition, Madhya Pradesh was a transit point for migrants travelling from Maharashtra and Gujarat to Uttar Pradesh, Bihar and other North East states. Consequently, a steep rise in the COVID-19 cases was seen in Ujjain after 10 April and this period was the toughest and the horrific. The situation worsened even after the government imposed a lockdown over the weekend on April 9 [12]. During this period, all the family members were getting infected which was the most panic-inducing thing. Among the four phases, transmission rate and disease induced death rate were the highest in this phase. Infected persons were showing the symptoms only after 6-7 days and the basic reproduction number had reached to its peak of 4.275. Health care system was collapsed due to rapidly increasing COVID-19 patients and acute shortage of oxygen, ICU beds, and ventilators. The second wave achieved its peak on 05 May by reporting 410 daily new confirmed cases.

Phase-IV. After causing havoc in the third phase, this wave started to calm down after mid-May. By 15 May, daily new confirmed cases were more than 250 and then started decreasing and became zero on 23 June. The disease parameters were almost same as Phase I. However, the incubation period was still 13-14 days. The basic reproduction number during this period also reduced to 2.302.

6. Conclusion

In the present work, we analyzed the spread of the coronavirus disease in Ujjain during the second wave. We studied an SEIR deterministic model and derived an associated dynamically consistent nonstandard finite difference

scheme. To determine the model parameters, we fitted the NSFD scheme to the data reported by the state government. For a deeper understanding, we divided the entire period under study into four phases and estimated the basic reproduction number for each phase. To check the relative importance of the model parameters, we performed the sensitivity analysis. The normalized sensitivity indices show similar trend in all phases that the disease prevalence mainly depends on the size of susceptible population, and transmission rate. However, it does not depend on how many exposed persons get their RTPCR test positive. Thus by restricting the exposed population, we can control the disease spread. Looking at the results of all the phases, it is clear that the maximum impact and spread of the disease occurred from mid-April to mid-May. In this duration, Ujjain reported 10,009 new cases, and the basic reproduction was 4.275. The present study confirms that any infectious disease spreads first in global cities and then gradually reaches smaller cities. In Ujjain too, COVID-19 was at its peak when the cases started decreasing in metro cities. Furthermore, the present study highlights the fact that the disease did not spread suddenly, but gradually increased its effect. This indicates that if people had become alert in advance and followed the rules of COVID-19 protocol like wearing masks and social distancing, then such a dreadful situation could have been avoided. We feel that if the lockdown had been placed a little earlier, the spread of the disease could have been reduced. The gist of the study is that if everyone protects themselves from coming in contact with an infected person, then everyone can help control the spread of the disease. The current model predicts satisfactory outcomes, yet some possible factors may influence the results. Classes of populations such as disease-induced death, hospitalization, quarantined, symptomatic, asymptomatic and so on were not included in the study. Some people have had corona tests in other cities as well and hence, the data of those people could not be included in the data of Ujjain district. The data of those who did not get their COVID tests done due to fear of isolation, quarantine or hospitalization could also not be included. By incorporating the above factors into the present study, one can better understand the disease prevalence.

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