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SINE MODI - G FAMILY OF DISTRIBUTIONS WITH APPLICATIONS

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Abstract

In this paper, we introduce a new family of probability distributions called the Modi - G family and derive its linear representation of the family. By applying the exponential distribution within this framework, we develop a specific case termed the Sine Modi exponential (SME) distribution. This model generalizes the exponential distribution, offering enhanced flexibility for modeling lifetime and survival data. We derive some fundamental properties of the SME distribution, including its probability density function, cumulative distribution function, survival function, hazard rate, quantile function, linear representation, moments, moment generating function and order statistics. Parameter estimation is performed using

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the method of maximum likelihood. To evaluate the applicability of the proposed model, three real-world datasets are analyzed. The findings demonstrate that the proposed model outperforms several existing models considered in the study.

1. Introduction

Probability distributions are essential tools in statistical modeling, especially for analyzing lifetime and survival data. In fields such as biomedicine, engineering, and reliability testing, precise modeling of time-to-event data is critical for understanding underlying mechanisms and supporting informed decisions. Although classical models like the exponential, Weibull, and log-logistic distributions are commonly employed, they often lack the flexibility to accommodate complex hazard rate behaviors seen in real-world data such as increasing, decreasing, bathtub-shaped, or inverted *J*-shaped patterns.

To address the shortcomings of classical probability models in capturing complex data patterns, numerous extended and generalized families of distributions have been developed. An example of this is the Burr-Hatke-G family proposed by Yousof et al. [21]. In the same vein, the generalized odd log-logistic introduced by Cordeiro et al. [4] and Yousof et al. [22] proposed that the extended odd Frechet distributions improve modeling versatility through the use of sophisticated transformation techniques. Additionally, families founded on the half-logistic distribution have greatly enhanced the evaluation of survival data, featuring prominent variants such as the Type I half-logistic developed by Cordeiro et al. [5] and the Type II log-logistic models proposed by Soliman et al. [17].

Recent developments in distribution theory have also focused on incorporating trigonometric transformations to improve model flexibility. For instance, distributions such as the arctan-X family of distributions introduced by Alkhairy et al. [3] and arctan family of distributions proposed by Gómez Déniz et al. [6] utilize arctangent-based transformations to modify standard distributions. Similarly, Sine-based models including the Sine-G developed by Mahmood et al. [11], Sine Topp-Leone-G family of distributions introduced by Al Babtain et al. [2], and Sine extended odd Frechet G family proposed by Jamal et al. [8] demonstrate the effectiveness of trigonometric generator functions in capturing diverse data characteristics across a range of applications.

Recent research has introduced several distribution families derived from the Modi and Sine-based approaches. For instance, Ndayisaba et al. [15] introduced the Modi exponentiated exponential distribution, whereas Modi and Singh [12] developed a new distribution model with applications in engineering confirmed through simulation methods. Kumawat et al. [10] proposed the Modi Weibull distribution, highlighting its inferential features and assessment through simulation. Within the framework of Sine-generated models, Oramulu et al. [16] introduced the Sine generalized family, detailing its essential characteristics, methods for parameter estimation, and practical significance. Isa et al. [7] proposed the Sine Type II Topp-Leone-G distribution, offering an extensive analysis of its mathematical structure and ability to fit empirical data. Additionally, Sapkota et al. [18] proposed a new member to the Sine-G class, showcasing its effectiveness in modeling medical datasets. Kumar et al. [9] and Souza et al. [20] proposed the Sine-G family by defining its cumulative distribution and probability density function as follows:

$$F(x) = \sin\left(\frac{\pi}{2} G(x)\right), \tag{1}$$

$$f(x) = \frac{\pi}{2} g(x) \cos\left(\frac{\pi}{2} G(x)\right). \tag{2}$$

Modi et al. [13] introduced the Modi family and its cdf and pdf defined as follows:

$$F(x) = \frac{(1+\alpha^{\beta})G(x)}{\alpha^{\beta} + G(x)}, \tag{3}$$

$$f(x) = \frac{(1+\alpha^{\beta})(\alpha^{\beta} g(x))}{\{\alpha^{\beta} + G(x)\}^2}, \quad x > 0,\ \alpha > 0,\ \beta > 0. \tag{4}$$

In this paper, we introduce a new family of distributions, termed the Sine Modi - G family, which is constructed by combining the Sine-G and Modi families (Section 2). In Section 3, we derive its linear representation. Building on this framework, we incorporate the exponential distribution in Section 4, leading to the development of the Sine Modi exponential distribution. Sections 5 and 6 present several of its mathematical properties. Parameter estimation method is discussed in Section 7, while the applicability of the proposed model is demonstrated in Section 8. Finally, the conclusion is provided in Section 9.

2. Sine Modi - G Family

The proposed Sine Modi - G family is defined by the cumulative distribution function (CDF) and probability density function (PDF) as

$$F(x) = \sin\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})G(x)}{\alpha^{\beta}+G(x)}\right), \tag{5}$$

$$f(x) = \frac{\pi}{2}\frac{(1+\alpha^{\beta})(\alpha^{\beta}g(x))}{\{\alpha^{\beta}+G(x)\}^{2}}\cos\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})G(x)}{\alpha^{\beta}+G(x)}\right), \quad x > 0,\ \alpha > 0,\ \beta > 0, \tag{6}$$

where $G(x)$ and $g(x)$ denote the baseline CDF and PDF, respectively.

3. Linear Representation of Sine Modi - G Family

One can derive useful linear expansions using exponentiated distributions, specifically the exponentiated-G (Exp-G) distribution with power parameter $z > 0$ which has the CDF:

$$G_z(x; \varphi) = [G(x; \varphi)]^{z}; \tag{7}$$

where $x \in R$. The corresponding PDF can be expressed as

$$g_z(x; \varphi) = zg(x)[G(x; \varphi)]^{z-1}; \quad x \in R. \tag{8}$$

These notations are used in the following discussion. Exponentiated distributions have well-known properties for a wide range of baseline CDF $G(x; \varphi)$. The linear representations of $F(x)$ and $f(x)$ in terms of Exp-G functions are shown in the following result. Using the Tayler expansion for trigonometric function Sin(x), the cumulative distribution function of Sine Modi - G family distribution can be expressed as

$$F^*(x) = \sum_{j=0}^{\infty} \Delta_j \left[\frac{G(x)}{\alpha^{\beta} + G(x)}\right]^{2j+1}, \tag{9}$$

where

$$\Delta_j = \frac{(-1)^j}{2j+1}\left(\frac{\pi}{2}(1+\alpha)^{\beta}\right)^{2j+1}.$$

The PDF corresponding to equation (9) can be calculated by differentiating it with respect to x; we obtain

$$f^*(x) = \sum_{j=0}^{\infty} \Delta_j^* g(x) \frac{[G(x)]^2}{(\alpha^{\beta} + G(x))^{2j+1}}, \tag{10}$$

where $\Delta_j^* = \Delta_j \alpha^{\beta}(2j+1)$.

4. Sine Modi Exponential Distribution

Let X be a continuous random variable. If the baseline distribution is exponential distribution, then the Sine Modi exponential distribution is defined as

$$F(x) = \sin\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})(1-e^{-\lambda x})}{\alpha^{\beta} + (1-e^{-\lambda x})}\right), \tag{11}$$

$$f(x) = \frac{\pi}{2}\frac{(1+\alpha^{\beta})(\alpha^{\beta}\lambda e^{-\lambda x})}{\{\alpha^{\beta} + (1-e^{-\lambda x})\}^2}\cos\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})(1-e^{-\lambda x})}{\alpha^{\beta} + (1-e^{-\lambda x})}\right), \tag{12}$$

where $x > 0$, $\alpha > 0$, $\beta > 0$ and $\lambda > 0$.

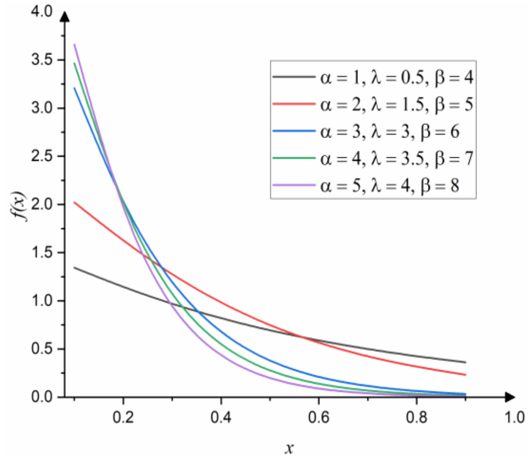

Figure 1. PDF plots of the Sine Modi exponential distribution.

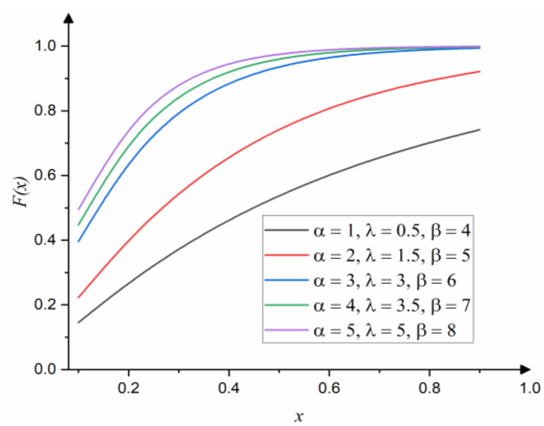

Figure 2. CDF plots of the Sine Modi exponential distribution.

5. Properties of the Sine Modi Exponential Distribution

5.1. Hazard rate function and survival function

The survival function and hazard rate function for the Sine Modi exponential distribution are given by

$$S(x) = 1 - \sin\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})(1-e^{-\lambda x})}{\alpha^{\beta}+(1-e^{-\lambda x})}\right), \tag{13}$$

$$H(x) = \frac{\frac{\pi}{2}\frac{(1+\alpha^{\beta})(\alpha^{\beta}\lambda e^{-\lambda x})}{\{\alpha^{\beta}+(1-e^{-\lambda x})\}^{2}}\cos\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})(1-e^{-\lambda x})}{\alpha^{\beta}+(1-e^{-\lambda x})}\right)}{1-\sin\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})(1-e^{-\lambda x})}{\alpha^{\beta}+(1-e^{-\lambda x})}\right)}. \tag{14}$$

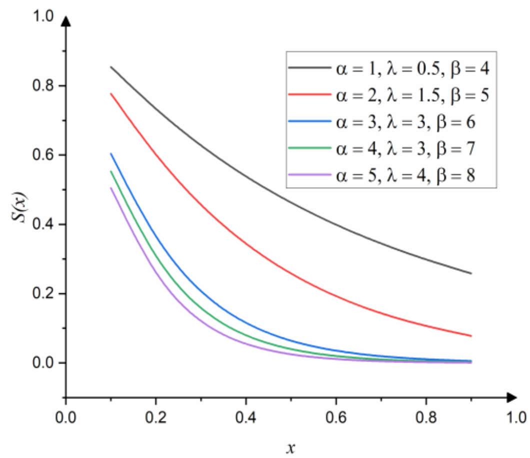

Figure 3. The survival function plots of the Sine Modi exponential distribution.

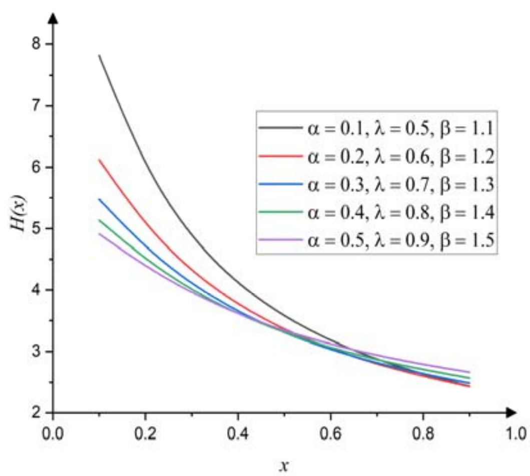

Figure 4. The hazard rate function plots of the Sine Modi exponential distribution.

5.2. Quantile function

The quantile function for the Sine Modi exponential distribution can be expressed as

$$Q(u) = -\frac{1}{\lambda}\left(\frac{1+\alpha^{\beta}-\left(\frac{2}{\pi}\sin^{-1}(u)(1+\alpha^{\beta})\right)}{1+\alpha^{\beta}-\left(\frac{2}{\pi}\sin^{-1}(u)\right)}\right). \tag{15}$$

5.3. Linear expansion of Sine Modi exponential distribution

Using equation (9), the expansion of the CDF defined in equation (11) is given by

$$F^{*}(x) = \sum_{j=0}^{\infty}\Delta_j\left[\frac{1-e^{-\lambda x}}{\alpha^{\beta}+1-e^{-\lambda x}}\right]^{2j+1}, \tag{16}$$

where $\Delta_j = \frac{(-1)^j}{2j+1}\left(\frac{\pi}{2}(1+\alpha)^{\beta}\right)^{2j+1}$. The PDF corresponding to equation (16) can be written as

$$f^{*}(x) = \sum_{j=0}^{\infty}\Delta_j^{**}e^{-\lambda x}\frac{(1-e^{-\lambda x})^{2j}}{(\alpha^{\beta}+1-e^{-\lambda x})^{2j+2}}, \tag{17}$$

where $\Delta_j^{**} = \Delta_j\lambda\alpha^{\beta}(2j+1)$.

5.4. Moments

The *r*th moment of random variable $X \sim SME(\alpha, \beta, \lambda)$ can be obtained by using the following expression:

$$\mu_r' = \sum_{j=0}^{\infty}\Delta_j^{**}\frac{\Gamma(r+1)}{\lambda^{r+1}(\alpha^{\beta}+1)^{2j+2}}\sum_{p=0}^{2j}\binom{2j}{p}(-1)^p$$

$$\times\sum_{m=0}^{\infty}\binom{2j+1+m}{2j+1}\frac{1}{(\alpha^{\beta}+1)^m}\frac{1}{(m+p+1)^{r+1}}. \tag{18}$$

5.5. Moment generating function of Sine Modi exponential distribution

The moment generating function of Sine Modi exponential distribution can be expressed as

$$M_X(t) = \sum_{j=0}^{\infty} \Delta_j^{**} \frac{1}{\lambda(\alpha^\beta + 1)^{2j+2}} \sum_{p=0}^{2j} \binom{2j}{p} (-1)^p$$

$$\times \sum_{m=0}^{\infty} \binom{2j+1+m}{2j+1} \frac{1}{(\alpha^\beta + 1)^m} \frac{1}{m + p - \frac{t}{\lambda}}. \quad (19)$$

6. Order Statistics

Let $X_1, X_2, \ldots, X_n$ be a random sample from the proposed distribution with $f(x)$ and $F(x)$. Then the PDF of the rth order statistic, denoted by X_r, is given by

$$f_{X_{(r)}} = \frac{n!}{(r-1)!(n-r)!} [F(x)]^{r-1} [1 - F(x)]^{n-r} f(x), \; r = 1, 2, 3, \ldots, n. \quad (20)$$

Substituting the expressions for $F(x)$ and $f(x)$, we get

$$f_{X_{(r)}} = \frac{n!}{(r-1)!(n-r)!} \frac{\pi}{2} \frac{(1+\alpha^\beta)(\alpha^\beta \lambda e^{-\lambda x})}{\{\alpha^\beta + (1 - e^{-\lambda x})\}^2} \cos\left(\frac{\pi}{2} \frac{(1+\alpha^\beta)(1 - e^{-\lambda x})}{\alpha^\beta + (1 - e^{-\lambda x})} \right)$$

$$\times \left[\sin\left(\frac{\pi}{2} \frac{(1+\alpha^\beta)(1 - e^{-\lambda x})}{\alpha^\beta + (1 - e^{-\lambda x})} \right) \right]^{r-1}$$

$$\times \left[1 - \sin\left(\frac{\pi}{2} \frac{(1+\alpha^\beta)(1 - e^{-\lambda x})}{\alpha^\beta + (1 - e^{-\lambda x})} \right) \right]^{n-r}. \quad (21)$$

Therefore, the probability density function of the smallest order statistics $X_{(1)}$ is given by

$$f_{X_{(r)}}(x) = n\left[1 - \sin\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})(1-e^{-\lambda x})}{\alpha^{\beta}+(1-e^{-\lambda x})}\right)\right]^{n-1} \frac{\pi}{2}\frac{(1+\alpha^{\beta})(\alpha^{\beta}\lambda e^{-\lambda x})}{\{\alpha^{\beta}+(1-e^{-\lambda x})\}^{2}}$$

$$\times \cos\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})(1-e^{-\lambda x})}{\alpha^{\beta}+(1-e^{-\lambda x})}\right) \tag{22}$$

and the probability density function of the largest order statistics $X_{(n)}$ is given by

$$f_{X_{(r)}}(x) = n\left[\sin\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})(1-e^{-\lambda x})}{\alpha^{\beta}+(1-e^{-\lambda x})}\right)\right]^{n-1} \frac{\pi}{2}\frac{(1+\alpha^{\beta})(\alpha^{\beta}\lambda e^{-\lambda x})}{\{\alpha^{\beta}+(1-e^{-\lambda x})\}^{2}}$$

$$\times \cos\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})(1-e^{-\lambda x})}{\alpha^{\beta}+(1-e^{-\lambda x})}\right). \tag{23}$$

7. Maximum Likelihood Estimation

By maximizing the log likelihood function, the maximum likelihood estimates of α, β and λ are obtained. In case of the Sine Modi exponential distribution, the log-likelihood function of the model is given by

$$\log L(\alpha, \beta, \lambda) = n\log\left(\frac{\pi}{2}\right) + n\log(1+\alpha^{\beta}) + n\log\lambda$$

$$+ \sum_{i=1}^{n}[\log(\alpha^{\beta}) - \lambda x_i - 2\log(\alpha^{\beta}+(1-e^{-\lambda x_i}))]$$

$$+ \log\left(\cos\left(\frac{\pi}{2}\frac{(1+\alpha^{\beta})(1-e^{-\lambda x_i})}{\alpha^{\beta}+(1-e^{-\lambda x_i})}\right)\right). \tag{24}$$

Partial derivative with respect to α,

$$\frac{\partial \log L}{\partial \alpha} = n\frac{\beta\alpha^{\beta-1}}{1+\alpha^{\beta}} + \sum_{i=1}^{n}\left\{\frac{\beta\alpha^{\beta-1}}{\alpha^{\beta}} - 2\frac{\beta\alpha^{\beta-1}}{\alpha^{\beta}+(1-e^{-\lambda x_i})} - \tan(c_i)\frac{\partial c_i}{\partial \alpha}\right\}, \tag{25}$$

where $c_i = \frac{\pi}{2}\frac{(1+\alpha^{\beta})(1-e^{-\lambda x_i})}{\alpha^{\beta}+(1-e^{-\lambda x_i})}$,

$$\frac{\partial c_i}{\partial \alpha} = \frac{[\beta\alpha^{\beta-1}(1-e^{-\lambda x_i})(\alpha^{\beta}+(1-e^{-\lambda x_i}))-(1+\alpha^{\beta})(1-e^{-\lambda x_i})\beta\alpha^{\beta-1}]}{(\alpha^{\beta}+(1-e^{-\lambda x_i}))^2}.$$

Partial derivative with respect to β,

$$\frac{\partial \log L}{\partial \beta} = n\frac{\alpha^{\beta}\log\alpha}{1+\alpha^{\beta}} + \sum_{i=1}^{n}\left[\frac{\log\alpha}{\alpha^{\beta}} - 2\frac{\alpha^{\beta}\log\alpha}{\alpha^{\beta}+(1-e^{-\lambda x_i})} - \tan(c_i)\frac{\partial c_i}{\partial \beta}\right], \quad (26)$$

where

$$\frac{\partial c_i}{\partial \beta} = \frac{\pi}{2}\frac{\alpha^{\beta}\log\alpha(1-e^{-\lambda x_i})(\alpha^{\beta}+(1-e^{-\lambda x_i}))-(1+\alpha^{\beta})(1-e^{-\lambda x_i})\alpha^{\beta}\log\alpha}{(\alpha^{\beta}+(1-e^{-\lambda x_i}))^2}.$$

Partial derivative with respect to λ,

$$\frac{\partial \log L}{\partial \lambda} = \frac{n}{\lambda} + \sum_{i=1}^{n}\left[-x_i - 2\frac{e^{-\lambda x_i}x_i}{\alpha^{\beta}+(1-e^{-\lambda x_i})} - \tan(c_i)\frac{\partial c_i}{\partial \lambda}\right], \quad (27)$$

where $\frac{\partial c_i}{\partial \lambda} = \frac{\pi}{2}\frac{(1+\alpha^{\beta})x_i e^{-\lambda x_i}\alpha^{\beta}}{(\alpha^{\beta}+(1-e^{-\lambda x_i}))^2}$.

Due to the complex nature of the log-likelihood, closed-form solutions for the MLEs are not available. Hence, numerical optimization techniques such as the Newton-Raphson method, BFGS, or Nelder-Mead algorithms are required to compute the estimates.

8. Applications

In this part of the work, we provide three application datasets to show the effectiveness and flexibility of the Sine Modi exponential distribution.

8.1. First data set

The first dataset consists of records of total annual rainfall (in inches) for the month of January from 1880 to 1916, measured at the Los Angeles Civic

Center. The values were provided in reference [19]. The data are as follows: 1.33, 1.43, 1.01, 1.62, 3.15, 1.05, 7.72, 0.20, 6.03, 0.25, 7.83, 0.25, 0.88, 6.29, 0.94, 5.84, 3.23, 3.7, 1.26, 2.64, 1.17, 2.49, 1.62, 2.1, 0.14, 2.57, 3.85, 7.02, 5.04, 7.27, 1.53, 6.70, 0.07, 2.01, 10.35, 5.42 and 13.3.

8.2. Second data set

The second data set is the failure times of eight components at three different temperatures, 100, 120, 140, introduced by [14]. The observed values are: 14.712, 32.644, 61.979, 65.521, 105.50, 114.60, 120.40, 138.50, 8.610, 11.741, 54.535, 55.047, 58.928, 63.391, 105.18, 113.02, 2.998, 5.016, 15.628, 23.040, 27.851, 37.843, 38.050 and 48.226.

8.3. Third data set

The third dataset is a real-world dataset comprising 50 observed device failure times, originally examined by Aarset [1]. The failure times (measured in arbitrary time units) are as follows: 0.1, 0.2, 1, 1, 1, 1, 1, 2, 3, 6, 7, 11, 12, 18, 18, 18, 18, 18, 21, 32, 36, 40, 45, 46, 47, 50, 55, 60, 63, 63, 67, 67, 67, 67, 72, 75, 79, 82, 82, 83, 84, 84, 84, 85, 85, 85, 85, 85, 86 and 86.

The Sine Modi exponential distribution is fitted to these three datasets and compared with the following:

- Modi exponential distribution:

$$f(x) = \frac{p\alpha^{\beta}e^{-px}}{\left(1 - \frac{e^{-px}}{1+\alpha^{\beta}}\right)^2 (1+\alpha^{\beta})}, \quad \alpha, \beta, p, x > 0.$$

- Sine - inverse Weibull:

$$f(x) = \frac{\pi}{2}\alpha\theta x^{-\theta-1}e^{-\alpha x^{-\theta}} \cos\left(\frac{\pi}{2}e^{-\alpha x^{-\theta}}\right), \quad x > 0, \alpha, \theta > 0.$$

- The inverse Weibull distribution:

$$f(x) = \frac{\tau(\theta/x)^{\tau} e^{-(\theta/x)^{\tau}}}{x}, \quad x > 0, \tau, \theta > 0.$$

- Weighted generalized quasi Lindley distribution (WGQLD):

$$f(x) = \frac{\theta^3 x^2(\theta^2 x^2 + 6\alpha\theta x + 6\alpha^2)e^{-\theta x}}{12(\alpha+1)(\alpha+2)}, \quad x > 0, \alpha, \theta > 0.$$

- Sine Burr XII distribution:

$$f(x) = \frac{\pi}{2}\frac{abx^{a-1}}{(1+x^a)^{b+1}}\cos\left\{\frac{\pi}{2}\left[1-\frac{1}{(1+x^a)^b}\right]\right\}, \quad a, b, x > 0.$$

The MLEs and corresponding log-likelihood $l(\cdot)$ values for Sine Modi exponential distribution for three datasets are provided in Table 1. For the decision about the best fitting of the competing model, we computed several criterion measures such as the Akaike information criterion (AIC), the corrected Akaike Information Criterion (AICc), the Bayesian information criterion (BIC), and the Hannan-Quinn information criterion (HQIC).

Table 1. The MLEs and corresponding log-likelihood $l(\cdot)$ values for the Sin Modi distribution

Data sets	Estimate	$l(x, \cdot)$
Dataset 1	$\alpha = 4.45164,\ \beta = 5.07621,\ \lambda = 0.16097$	–83.24986
Dataset 2	$\alpha = 9.81765,\ \beta = 9.81765,\ \lambda = 0.01015$	–119.8293
Dataset 3	$\alpha = 2.544539,\ \beta = 5.025881,\ \lambda = 0.012121$	–240.624

From the results given in Tables 2, 3 and 4, we noted that the Sine Modi exponential distribution provides a better fit with the minimum value of AIC, AICc, BIC, HQIC and K-S and the largest *p*-values compared with other models considered in this work.

Table 2. The goodness of fit tests for Dataset 1

Distributions	AIC	AICc	BIC	HQIC	K-S	*p*-value
Sine Modi exponential	**172.4997**	**173.2007**	**177.3062**	**174.1772**	**0.0906**	**0.9214**
Modi exponential	174.6578	175.385	179.4905	176.3615	0.7884	2.2e-16
Sine-inverse Weibull	184.3137	184.6666	187.5355	185.4495	0.1586	0.3096
Inverse Weibull	190.8537	191.2066	194.0755	191.9896	0.1897	0.1394
WGQLD	206.7907	207.1436	210.0125	207.9265	0.2682	0.0097
Sine Burr XII	181.3963	181.7493	184.6181	182.5322	0.1423	0.4417

Table 3. The goodness of fit tests for Dataset 2

Distributions	AIC	AICc	BIC	HQIC	K-S	*p*-value
Sine Modi exponential	**245.6586**	**246.8586**	**249.1927**	**246.5962**	**0.1192**	**0.8454**
Modi exponential	246.4595	247.6595	249.9937	247.3971	0.12815	0.7794
Sine-inverse Weibull	251.1870	251.7585	253.5431	251.8121	0.1546	0.5622
Inverse Weibull	255.0592	255.6306	257.4153	255.6843	0.1778	0.3884
WGQLD	252.8124	253.3839	255.1686	253.4375	0.1950	0.2824
Sine Burr XII	284.8518	285.4232	287.2079	285.4768	0.3609	0.0026

Table 4. The goodness of fit tests for Dataset 3

Distributions	AIC	AICc	BIC	HQIC	K-S	*p*-value
Sine Modi exponential	**487.248**	**487.7697**	**492.984**	**489.4323**	**0.18794**	**0.0584**
Modi exponential	488.1792	488.7009	493.9153	490.3635	0.1911	0.05189
Sine-inverse Weibull	523.2495	523.5048	527.0736	524.7057	0.25731	0.0026
Inverse Weibull	534.0281	534.2834	537.8521	535.4843	0.57414	9.659e-15
WGQLD	574.4734	574.7287	578.2975	575.9296	0.23662	0.0074
Sine Burr XII	544.5377	544.793	548.3617	545.9939	0.32766	4.349e-05

9. Conclusion

In this study, we proposed a new family of probability distributions, namely the Sine Modi - G family, and derived its linear transformation of the family. A special case, the Sine Modi exponential (SME) distribution, was introduced by embedding the exponential distribution within this framework. Several structural properties of the SME distribution were derived, including its probability density and cumulative distribution functions, survival and hazard rate functions, quantile function, linear representation, moments, moment-generating function, and order statistics. The parameters of the SME distribution were estimated using the method of maximum likelihood. To demonstrate the practical relevance of the proposed model, three real-world datasets were analyzed. The empirical results revealed that the SME distribution provides a superior fit compared to several existing competing models, thereby establishing its effectiveness in modeling lifetime and survival data. This study highlights the flexibility of the SME distribution and its potential applications in reliability analysis, medical studies, and other fields involving time-to-event data. Future research may extend this work by exploring Bayesian estimation methods, regression modeling under the SME framework.

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