**Advances and Applications in Statistics**
© 2026 Pushpa Publishing House, Prayagraj, India
https://pphmjopenaccess.com
https://doi.org/10.17654/0972361726002 P-ISSN: 0972-3617
Volume 93, Number 1, January 2026, Pages 19-32 E-ISSN: 3049-2165

# NEW NONPARAMETRIC TESTS FOR LOCATION-SCALE TESTING IN A MIXED DESIGN

**Fouad Khalawi**

Department of Statistics
Faculty of Science
King Abdulaziz University
Jeddah 21589, Saudi Arabia
e-mail: fakhlwi@kau.edu.sa

## Abstract

A mixed experimental design that combines Randomized Complete Block Design (RCBD) and Completely Randomized Design (CRD) can be structured to get advantages from both types of designs. In some experiments, researchers start with an RCBD plan but they might need to switch to a CRD due to circumstances beyond their control. Data from a mixed design can be analyzed using statistical methods that combine different statistics. In this paper, new tests are developed for testing the differences in mean and variances simultaneously for mixed designs of RCBD and CRD using nonparametric methods. These nonparametric tests are estimated and compared under a symmetric distribution via Monte Carlo simulation.

Received: October 10, 2025; Revised: October 15, 2025; Accepted: December 23, 2025
2020 Mathematics Subject Classification: 62G10.
Keywords and phrases: Levene transformation, simple tree alternative, location-scale problem.

How to cite this article: Fouad Khalawi, New nonparametric tests for location-scale testing in a mixed design, Advances and Applications in Statistics 93(1) (2026), 19-32.
https://doi.org/10.17654/0972361726002

Published Online: January 2, 2026

## 1. Introduction

In certain experiments, workers begin with a particular design plan but they might need to switch to a different design due to circumstances beyond their control before the experiment is completed. This scenario could occur when comparing two treatments in the two-sample studies. For example, when two treatments are applied to the same subject, paired data is generated. However, if some subjects cannot receive one of the treatments, the result is a mix of paired and unpaired data. Since the 1970s, multiple techniques have been proposed for combining paired and unpaired data. Early contributions in this area were made by Morrison [1], Lin and Stivers [2], Ekbohm [3], and Bhoj [4]. Lately, Dubnicka et al. [5] gathered two nonparametric statistics utilizing the Wilcoxon signed-rank statistic for paired data and the Wilcoxon rank-sum statistic for unpaired. Magel and Fu [6] later modulated the unweighted statistic proposed by Dubnicka et al. [5].

Recently, multiple tests have been proposed for combining different designs under various alternative hypotheses. For instance, when the data are a mixture of an RCBD and CRD, Magel et al. [7] developed a test that combined the Friedman test and Kruskal-Wallis test for testing the equality of *K*-median under the general alternative. For an umbrella alternative, another test was developed by combining the standardized version of the Mack-Wolfe test and the Kim-Kim test. In the case of the simple tree alternative, Olet and Magel [8] introduced a nonparametric test for testing location change for the mixed design of RCBD and CRD. These tests combine the Fligner-Wolfe test and the modified Page's test. Magel and Ndungu [9] developed a new test by combining the Jonckheere-Terpstra test and the Alvo and Cabilio test for a mixed design of CRD and a randomized block design (RBD) under the nondecreasing alternative.

For location and scale testing, Lepage [10] combined the Wilcoxon rank-sum and Ansari-Bradley test statistics to detect the location and scale changes. Neuhauser and Hothorn [11] proposed location-scale tests based on a combination of Bartholomen's test statistics for location trends and

Bartholomen's test statistics for scale trends, where the Levene transformation was applied to the scale trend test. Neuhauser et al. [12] introduced a permutation test based on Levene's idea for location-scale testing. The test is the sum of standardized Wilcoxon rank sum test statistics for location change and standardized Wilcoxon rank sum test statistics after the original observations undergo permutation for scale change. Levene [13] proposed an approach testing the differences in variances using transformed data. The transformation is calculated by subtracting each observation from its sample mean and then taking the absolute value. Brown and Forsythe [14] expanded upon Levene's test by incorporating the option to use either the median or the trimmed mean in addition to the mean. They recommended using the median for asymmetric underlying distribution and the trimmed mean for heavy-tiled underlying distribution.

This paper proposes a test for a mixed design using Levene's transformation. The probability of type I error and the power of the new test will be estimated and compared with other tests.

## 2. Materials and Methods

Khalawi and Magel [15, 16] introduced six nonparametric tests for mean and variance testing in a mixed design of RCBD and CRD for symmetric and asymmetric distributions. These tests combine the Fligner-Wolfe test (FW), modified Page's test (MP), and modified Ansari-Bradley test for CRD (AB1) and RCBD (AB2). Some of these tests are standardized before being combined and then re-standardized, such as the first test $Z_1$ while other tests are combined the test statistics first and then standardized, such as the second test $Z_2$. Moreover, they add weight to previous tests to get other tests like the third, fourth, fifth, and sixth tests. These tests are given below:

$$Z_1 = \frac{(Z_{FW} + Z_{MP} + Z_{AB1} + Z_{AB2})}{\sqrt{(4)}}, \tag{1}$$

$$Z_2 = \frac{(T_{FW} + T_{MP} + T_{AB1} + T_{AB2}) - (E(T_{FW}) + E(T_{MP}) + E(T_{AB1}) + E(T_{AB2}))}{\sqrt{\mathrm{Var}(T_{FW}) + \mathrm{Var}(T_{MP}) + \mathrm{Var}(T_{AB1}) + \mathrm{Var}(T_{AB2})}}, \tag{2}$$

$$Z_3 = \frac{\left(\frac{n_a}{n} Z_{FW} + \frac{n_b}{n} Z_{MP} + \frac{n_a}{n} Z_{AB1} + \frac{n_b}{n} Z_{AB2}\right)}{\sqrt{\left(\frac{n_a^2}{n^2} + \frac{n_b^2}{n^2} + \frac{n_a^2}{n^2} + \frac{n_b^2}{n^2}\right)}}, \tag{3}$$

$$Z_4 = \frac{\left(\frac{n_a}{n} T_{FW} + \frac{n_b}{n} T_{MP} + \frac{n_a}{n} T_{AB1} + \frac{n_b}{n} T_{AB2}\right) - \left(\frac{n_a}{n} E(T_{FW}) + \frac{n_b}{n} E(T_{MP}) + \frac{n_a}{n} E(T_{AB1}) + \frac{n_b}{n} E(T_{AB2})\right)}{\sqrt{\frac{n_a^2}{n^2} Var(T_{FW}) + \frac{n_b^2}{n^2} Var(T_{MP}) + \frac{n_a^2}{n^2} Var(T_{AB1}) + \frac{n_b^2}{n^2} Var(T_{AB2})}}, \tag{4}$$

$$Z_5 = \frac{\left(\frac{n_b}{n} Z_{FW} + \frac{n_a}{n} Z_{MP} + \frac{n_b}{n} Z_{AB1} + \frac{n_a}{n} Z_{AB2}\right)}{\sqrt{\left(\frac{n_b^2}{n^2} + \frac{n_a^2}{n^2} + \frac{n_b^2}{n^2} + \frac{n_a^2}{n^2}\right)}}, \tag{5}$$

$$Z_6 = \frac{\left(\frac{n_b}{n} T_{FW} + \frac{n_a}{n} T_{MP} + \frac{n_b}{n} T_{AB1} + \frac{n_a}{n} T_{AB2}\right) - \left(\frac{n_a}{n} E(T_{FW}) + \frac{n_a}{n} E(T_{MP}) + \frac{n_a}{n} E(T_{AB1}) + \frac{n_a}{n} E(T_{AB2})\right)}{\sqrt{\frac{n_b^2}{n^2} \mathrm{Var}(T_{FW}) + \frac{n_a^2}{n^2} \mathrm{Var}(T_{MP}) + \frac{n_b^2}{n^2} \mathrm{Var}(T_{AB1}) + \frac{n_a^2}{n^2} \mathrm{Var}(T_{AB2})}}, \tag{6}$$

here, $n$ shows the total sample size, which is the sum of the sample size for each treatment under the CRD portion $n_a$ and the number of blocks under the RCBD portion $n_b$. Under $H_0$, all of the proposed tests follow an asymptotic standard normal distribution, and the null hypothesis is rejected for large values. The recommendation overall was to use $Z_1$ or $Z_3$ for symmetric distributions.

This paper aims to present two statistical tests for location and scale problems in a mixed design. The mixed design structure for these tests combines RCBD and CRD. These tests are used to assess the null hypothesis of no differences between the treatments and the control, against a one-sided alternative that at least one treatment is greater than the control, utilizing a single distribution-free test (simple tree alternative). Let $K$ populations given a sample $X_{ij}$, where $i = 1, 2, ..., n_j$ and $j = 1, 2, ..., K$. Then the null and alternative hypotheses for this work can be written as:

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_k, \text{ and } H_0 : \sigma_1 = \sigma_2 = \cdots = \sigma_k$$

$$H_a : \mu_1 \le [\mu_2, \mu_3, ..., \mu_k] \text{ and/or } H_a : \sigma_1 \le [\sigma_2, ..., \sigma_K]$$

with at least one strict inequality,

here $\mu_i$ represents the location parameter of population $i$ and $\sigma_i$ denotes the scale parameter of population $i$. Population one $i = 1$ is the control population, while populations 2 through $k$ represent the treatment populations.

Two test statistics are proposed for the mixed design including an RCBD and CRD. The proposed test statistics are combinations of the Fligner-Wolfe test (FW) and modified Page's test (MP). Let $Z_{FW}$ and $Z_{MP}$ denote the standardized versions of the FW and MP tests, respectively. The FW test will be applied to the CRD portion, while the MP test will be used for the RCBD portion. Because the objective is to detect both location and scale changes, the FW and MP tests are each computed twice: once using the original observations to detect location changes, and once using transformed observations to detect scale changes. We suggest transforming the original observation using the Levene approach, which is the absolute deviation of the original outcomes $X_{ij}$ from their group means $\bar{x}_i$ $(Y_{ij} = | X_{ij} - \bar{x}_i |, i = 1, 2, ..., n_j, j = 1, 2, ..., K)$. The first proposed test is given by

$$Q_1 = \frac{(Z_{FW}(L) + Z_{MP}(L) + Z_{FW}(S) + Z_{MP}(S))}{\sqrt{(4)}}$$

$$= \frac{\left( \frac{T_{FW(L)} - \frac{n_t(N+1)}{2}}{\sqrt{\frac{n_c n_t(N+1)}{12}}} + \frac{T_{MP(L)} - n_b\left(k^2 + \frac{k-1}{2}\right)}{\sqrt{n_b\left(\frac{k^2-1}{12}\right)}} + \frac{T_{FW(S)} - \frac{n_t(N+1)}{2}}{\sqrt{\frac{n_c n_t(N+1)}{12}}} + \frac{T_{MP(S)} - n_b\left(k^2 + \frac{k-1}{2}\right)}{\sqrt{n_b\left(\frac{k^2-1}{12}\right)}} \right)}{\sqrt{(4)}}, \quad (7)$$

where

- $Z_{FW}(L)$ is the standardized version of FW calculated from the original observation $(X_{ij})$.
- $Z_{FW}(S)$ is the standardized version of FW calculated from the transformed observation $(Y_{ij})$.
- $Z_{MP}(S)$ is the standardized version of MP calculated from the original observation $(X_{ij})$.
- $Z_{MP}(L)$ is the standardized version of MP calculated from the transformed observation $(Y_{ij})$.
- $T_{FW(L)}$ is the Fligner-Wolfe test statistic calculated from the original observation $(X_{ij})$.
- $T_{FW(S)}$ is the Fligner-Wolfe test statistic calculated from the transformed observation $(Y_{ij})$.
- $T_{MP(L)}$ is modified Page's test statistic calculated from the original observation $(X_{ij})$.

- $T_{MP(S)}$ is modified Page's test statistic calculated from the transformed observation $(Y_{ij})$.

- $n_c$ is the number of observations in the control sample.

- $n_t$ is the number of observations in the combined treatment sample.

- $N = n_c + n_t$.

- $n_b$ is the number of blocks.

- $K$ is the number of treatments.

Let E(FW) and E(MP) represent the means of the FW and MP tests, respectively, while Var(FW) and Var(MP) represent their variances. The second proposed test is defined as follows:

$$Q_2 = \frac{\begin{array}{l}(T_{FW(L)} + T_{MP(L)} + T_{FW(S)} + T_{MP(S)}) \\ -\,(E(T_{FW(L)}) + E(T_{MP(L)}) + E(T_{FW(S)}) + E(T_{MP(S)}))\end{array}}{\sqrt{\mathrm{Var}(T_{FW(L)}) + \mathrm{Var}(T_{MP(L)}) + \mathrm{Var}(T_{FW(S)}) + \mathrm{Var}(T_{MP(S)})}}$$

$$= \frac{\begin{array}{l}(T_{FW(L)} + T_{MP(L)} + T_{FW(S)} + T_{MP(S)}) \\ -\left(\frac{n_t(N+1)}{2} + n_b\left(k^2 + \frac{k-1}{2}\right) + \frac{n_t(N+1)}{2} + n_b\left(k^2 + \frac{k-1}{2}\right)\right)\end{array}}{\sqrt{\frac{n_c n_t(N+1)}{12} + n_b\left(\frac{k^2-1}{12}\right) + \frac{n_c n_t(N+1)}{12} + n_b\left(\frac{k^2-1}{12}\right)}}. \quad (8)$$

Under the null hypothesis, the asymptotic distribution of both the first and second proposed tests follows a standard normal distribution. The null hypothesis is rejected for large values.

## 3. Simulation Study

The study used the Monte Carlo simulation to estimate and compare the power of the two proposed tests and the power of the Khalawi and Magel tests [15]. The simulations were implemented using SAS 9 for all programming [17]. The simple tree alternative, where the first population represents the control and the rest will be the other treatments, was used in this study. The probability of type one error for each test statistic was calculated and compared to the standard value of significant level 0.05. The probability of type one error for each test was computed as the proportion of times the null hypothesis was rejected under $H_0$ across all simulated datasets which was 10,000 samples. The power was evaluated as the proportion of times the null hypothesis was correctly rejected under $H_a$ based on 10,000 samples.

In this work, we considered symmetric distributions including the normal distribution and *t*-distribution with 3 degrees of freedom $DF = 3$. We considered a range of scenarios involving equal and unequal arrangements of sample sizes and number of blocks, as well as different location and scale parameter arrangements. We also considered different numbers of populations and both equal and unequal variance ratios between the CRD and RCBD portions.

For three populations, it included scenarios where the second and third populations shared identical parameters different from the first, where the first and second populations shared parameters different from the third, and where all populations had varied and unequally spaced parameters. With four populations, it considered setups where the last three populations shared parameters different from the first, where the first and second populations shared parameters different from the third and fourth, and where all populations had diverse and unequally spaced parameters. Lastly, for five populations, it examined cases where the last four populations shared parameters different from the first, where the first and second populations shared parameters different from the last three, and where all populations had varied and unequally spaced parameters.

## 4. Results and Discussion

The type I error rates and powers for the Khalawi and Magel tests $(Z_1$ to $Z_6)$ and the new tests $(Q_1$ and $Q_2)$ are summarized in Table 1. These results are detailed for different combinations of treatment effects (parameter arrangements), number of blocks in the RCBD portion, sample sizes in the CRD portion, and number of populations (*K*) under symmetric distributions: normal distribution (*N*), and *t*-distribution with $DF = 3$ (*T*).

When the sample size and blocks are 10, and the number of population is 3 under normal distribution, for scenarios with no treatment effect $(0, 1)$ $(0, 1)$ $(0, 1)$, the type I error rates are similar across all tests. Both the Khalawi and Magel tests and the new tests have values around 0.05. In cases of moderate treatment effect such as $(0, 1)$ $(1, 2)$ $(1, 2)$, the new tests show higher powers $(0.9590$ for $Q_1$ and 0.8636 for $Q_2)$ compared to the Khalawi and Magel tests, which range around 0.7473 for $Z_1$, $Z_3$, and $Z_5$, and 0.6964 for $Z_2$, $Z_4$, and $Z_6$. When treatment effects are higher with unequal space something like $(0, 1)$ $(1.5, 6)$ $(1.75, 8)$, the new tests show higher powers $(0.9996$ for $Q_1$ and 0.9869 for $Q_2)$ compared to the Khalawi and Magel tests $(0.8951$ for $Z_1$, $Z_3$, and $Z_5$, and 0.7417 for $Z_2$, $Z_4$, and $Z_6)$.

For a smaller sample size of 5, with 10 blocks and $k = 5$, under no treatment effect $(0, 1)$ $(0, 1)$ $(0, 1)$ $(0, 1)$ $(0, 1)$, the type I error rates maintain the alpha value which is 0.05 for the Khalawi and Magel tests and the new tests. In the case of the last four treatment effects have the same location and scale parameters that are different than the first treatment effect $(0, 1)$ $(1, 2)$ $(1, 2)$ $(1, 2)$ $(1, 2)$, the new tests show higher powers $(0.9563$ for $Q_1$ and 0.9145 for $Q_2)$ compared to the Khalawi and Magel tests, which show values ranging from 0.6006 to 0.7862. For higher treatment effects $(0, 1)$ $(0.5, 2.5)$ $(1, 5)$ $(1.5, 7.5)$ $(2, 10)$, the new tests have nearly the highest powers $(0.9986$ for $Q_1$ and 0.9925 for $Q_2)$, whereas the Khalawi and Magel tests show powers that range from 0.7450 to 0.9623.

When the sample size is 10, with 5 blocks, and $k = 3$, under $T$ distribution with $\sigma = \sqrt{3}$, for no treatment effect $(0, \sqrt{3})$ $(0, \sqrt{3})$ $(0, \sqrt{3})$, the type I error rates are slightly higher for the new tests (around 0.058) compared to the Khalawi and Magel tests (around 0.049). With a moderate treatment effect $(0, \sqrt{3})$ $(1, 2\sqrt{3})$ $(1, 2\sqrt{3})$, the new tests show higher powers $(0.7807$ for $Q_1$ and 0.7160 for $Q_2)$ compared to the Khalawi and Magel tests (ranging from 0.4201 to 0.6093).

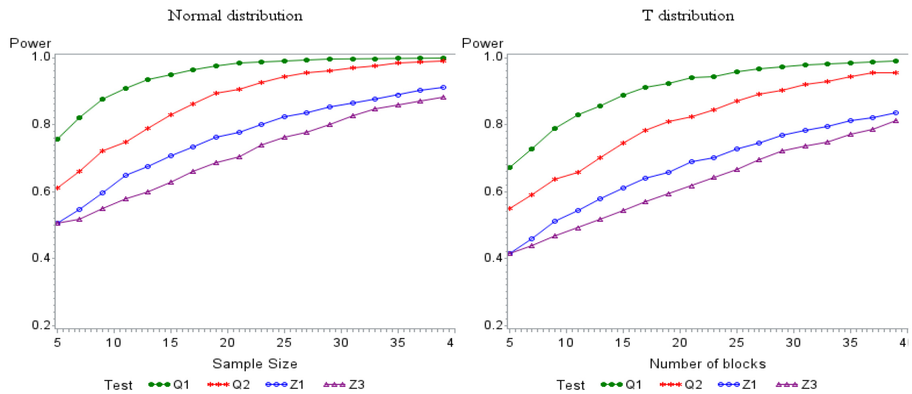
For a sample size of 10, with 10 blocks and $k = 4$, under no treatment effect $(0, \sqrt{3})$ $(0, \sqrt{3})$ $(0, \sqrt{3})$ $(0, \sqrt{3})$, the type I error rates maintain the alpha value across all tests, with values around 0.052 for the Khalawi and Magel tests and around 0.057 for the new tests. For moderate treatment effects $(0, \sigma)$ $(1, 2\sqrt{3})$ $(1, 2\sqrt{3})$ $(1, 2\sqrt{3})$, the new tests have higher powers $(0.9127$ for $Q_1$ and 0.7983 for $Q_2)$ than Khalawi and Magel tests.

Figure 1 presents the powers of the tests under normal and $T$ distributions when the variance in the CRD portion is larger than the variance in the RCBD portion under a function of sample sizes and number of blocks, respectively. In the left panel of Figure 1, which illustrates the normal distribution, the powers increase with the sample size for all tests. The new tests $(Q_1$ and $Q_2)$ consistently exceed the Khalawi and Magel tests $(Z_1$ and $Z_3)$, achieving near-perfect power as the sample size approaches 40. The right panel of Figure 1, showing the $T$ distribution, illustrates that the power increases with the number of blocks for all tests. Here, the new tests again demonstrate higher power, particularly $Q_1$.

Overall, the new tests $(Q_1$ and $Q_2)$ show consistently higher power compared to the Khalawi and Magel tests $(Z_1$ to $Z_6)$ across various scenarios. This suggests that the new tests are more robust in detecting location and scale changes under symmetric distributions.

**Table 1.** Type one error rate and power for symmetric distributions

| | Khalawi and Magel Tests | | | | | | New Tests | |
|---|---|---|---|---|---|---|---|---|
| Treatments Effects $(\mu,\sigma)$ | $Z_1$ | $Z_2$ | $Z_3$ | $Z_4$ | $Z_5$ | $Z_6$ | $Q_1$ | $Q_2$ |
| The sample size is 10, and the number of blocks is 10; k=3; N | | | | | | | | |
| (0,1) (0,1) (0,1) | 0.0542 | 0.0531 | 0.0542 | 0.0531 | 0.0542 | 0.0531 | 0.0562 | 0.0558 |
| (0,1) (1,2) (1,2) | 0.7473 | 0.6964 | 0.7473 | 0.6964 | 0.7473 | 0.6964 | 0.9590 | 0.8636 |
| (0,1) (0,1) (1,3) | 0.4246 | 0.2782 | 0.4246 | 0.2782 | 0.4246 | 0.2782 | 0.5447 | 0.4007 |
| (0,1) (0,2) (1,3) | 0.6235 | 0.4222 | 0.6235 | 0.4222 | 0.6235 | 0.4222 | 0.8652 | 0.7206 |
| (0,1) (0.5,2.5) (1,5) | 0.7667 | 0.5754 | 0.7667 | 0.5754 | 0.7667 | 0.5754 | 0.9770 | 0.8964 |
| (0,1) (0.25,2) (0.5,5) | 0.6427 | 0.4130 | 0.6427 | 0.413 | 0.6427 | 0.4130 | 0.9041 | 0.7716 |
| (0,1) (1,3) (1.5,3.5) | 0.8550 | 0.7487 | 0.8550 | 0.7487 | 0.855 | 0.7487 | 0.9952 | 0.9601 |
| (0,1) (1.5,6) (1.75,8) | 0.8951 | 0.7417 | 0.8951 | 0.7417 | 0.8951 | 0.7417 | 0.9996 | 0.9869 |
| The sample size is 5, and the number of blocks is 10; k=5; N | | | | | | | | |
| (0,1) (0,1) (0,1) (0,1) (0,1) | 0.0482 | 0.0429 | 0.0446 | 0.0443 | 0.0522 | 0.0460 | 0.0544 | 0.0498 |
| (0,1) (1,2) (1,2) (1,2) (1,2) | 0.7414 | 0.7332 | 0.7862 | 0.7226 | 0.6006 | 0.7655 | 0.9563 | 0.9145 |
| (0,1) (0,1) (0,3) (1,3) (1,3) | 0.6495 | 0.4282 | 0.7036 | 0.4208 | 0.5013 | 0.4596 | 0.8005 | 0.7064 |
| (0,1) (0,2) (1,3) (1,4) (1,5) | 0.8435 | 0.6383 | 0.8975 | 0.6281 | 0.6811 | 0.6686 | 0.9739 | 0.9375 |
| (0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10) | 0.9271 | 0.7546 | 0.9623 | 0.7450 | 0.7814 | 0.7840 | 0.9986 | 0.9925 |
| (0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5) | 0.7460 | 0.5697 | 0.7999 | 0.5615 | 0.5913 | 0.6041 | 0.9018 | 0.8324 |
| (0,1) (1,3) (1.5,3.5) (2,2) (2.5,6) | 0.8918 | 0.8779 | 0.9202 | 0.8689 | 0.7616 | 0.8998 | 0.9997 | 0.9973 |
| (0,1) (1,4) (1.5,6) (1.75,8) (2.5,10) | 0.9479 | 0.8311 | 0.9738 | 0.8259 | 0.8269 | 0.8526 | 0.9998 | 0.9990 |
| The sample size is 10, and the number of blocks is 5; k=3; T; $\sigma=\sqrt{3}$ | | | | | | | | |
| $(0,\sqrt{3})$ $(0,\sqrt{3}$ $(0,\sqrt{3})$ | 0.0518 | 0.0493 | 0.0497 | 0.0488 | 0.0561 | 0.0491 | 0.0585 | 0.0582 |
| $(0,\sqrt{3})$ $(1,2\sqrt{3})$ $(1,2\sqrt{3})$ | 0.5511 | 0.5530 | 0.6093 | 0.5348 | 0.4201 | 0.5757 | 0.7807 | 0.7160 |
| $(0,\sqrt{3})$ $(0,\sqrt{3})$ $(0,3\sqrt{3})$ | 0.3025 | 0.2363 | 0.3321 | 0.2271 | 0.2402 | 0.2448 | 0.3896 | 0.3301 |
| $(0,\sqrt{3})$ $(0,2\sqrt{3})$ $(1,3\sqrt{3})$ | 0.4713 | 0.3505 | 0.5364 | 0.3390 | 0.3485 | 0.3619 | 0.6667 | 0.5986 |
| $(0,\sqrt{3})$ $(0.5,2.5\sqrt{3})$ $(1,5\sqrt{3})$ | 0.6161 | 0.4783 | 0.7068 | 0.4662 | 0.4448 | 0.4963 | 0.8663 | 0.7965 |
| $(0,\sqrt{3})$ $(0.25,2\sqrt{3})$ $(0.5,5\sqrt{3})$ | 0.4888 | 0.3423 | 0.5684 | 0.3310 | 0.3540 | 0.3555 | 0.7395 | 0.6667 |
| $(0,\sqrt{3})$ $(1,3\sqrt{3})$ $(1.5,3.5\sqrt{3})$ | 0.7008 | 0.6466 | 0.7798 | 0.6313 | 0.5284 | 0.6696 | 0.9279 | 0.8836 |
| $(0,\sqrt{3})$ $(1.5,6\sqrt{3})$ $(1.75,8\sqrt{3})$ | 0.8017 | 0.6736 | 0.8876 | 0.6622 | 0.6012 | 0.6914 | 0.9859 | 0.9668 |
| The sample size is 10, and the number of blocks is 10; k=4; T; $\sigma=\sqrt{3}$ | | | | | | | | |
| $(0,\sqrt{3})$ $(0,\sqrt{3})$ $(0,\sqrt{3})$ $(0,\sqrt{3})$ | 0.0526 | 0.048 | 0.0526 | 0.0480 | 0.0526 | 0.0480 | 0.0574 | 0.0588 |
| $(0,\sqrt{3})$ $(1,2\sqrt{3})$ $(1,2\sqrt{3})$ $(1,2\sqrt{3})$ | 0.7076 | 0.606 | 0.7076 | 0.6060 | 0.7076 | 0.6060 | 0.9127 | 0.7983 |
| $(0,\sqrt{3})$ $(0,\sqrt{3})$ $(0,3\sqrt{3})$ $(1, 3\sqrt{3})$ | 0.4688 | 0.2566 | 0.4688 | 0.2566 | 0.4688 | 0.2566 | 0.6379 | 0.4911 |
| $(0,\sqrt{3})$ $(0,2\sqrt{3})$ $(1,3\sqrt{3})$ $(1,4\sqrt{3})$ | 0.7446 | 0.4874 | 0.7446 | 0.4874 | 0.7446 | 0.4874 | 0.9277 | 0.8061 |
| $(0,\sqrt{3})$ $(0.5,2.5\sqrt{3})$ $(1,5\sqrt{3})(1.5,7.5\sqrt{3})$ | 0.8728 | 0.6220 | 0.8728 | 0.6220 | 0.8728 | 0.622 | 0.9907 | 0.9462 |
| $(0,\sqrt{3})$ $(0.25,2\sqrt{3})$ $(0.5,5\sqrt{3})$ $(0.75,\sqrt{3})$ | 0.6235 | 0.4018 | 0.6235 | 0.4018 | 0.6235 | 0.4018 | 0.7708 | 0.6152 |
| $(0,\sqrt{3})$ $(1,3\sqrt{3})$ $(1.5,3.5\sqrt{3})$ $(2,2\sqrt{3})$ | 0.8701 | 0.7783 | 0.8701 | 0.7783 | 0.8701 | 0.7783 | 0.9904 | 0.9563 |
| $(0,\sqrt{3})$ $(1,4\sqrt{3})$ $(1.5,6\sqrt{3})$ $(1.75,8\sqrt{3})$ | 0.9274 | 0.7316 | 0.9274 | 0.7316 | 0.9274 | 0.7316 | 0.9986 | 0.9861 |




(a) (b)

**Figure 1.** Estimated power when the variance of the CRD is greater than the variance of the RCBD. (a) Increasing the sample size in the CRD portion; (b) Increasing the number of blocks in the RCBD portion.

## 5. Conclusions

This work proposes two test statistic versions to use with mixed design data when three or more treatments are introduced under a simple tree alternative. The comparison between the Khalawi and Magel tests $(Z_1$ to $Z_6)$ and the new tests $(Q_1$ and $Q_2)$ shows that these new tests generally perform better across various scenarios. The new tests consistently demonstrate higher powers than their Khalawi and Magel tests. This improved power is evident across different sample sizes and block numbers, as illustrated by the results summarized in Table 1. When the variance in the CRD portion is larger than the variance in the RCBD portion, as the graphical analysis presented in Figure 1, the new tests $(Q_1$ and $Q_2)$ again have higher power compared to the Khalawi and Magel tests. The type I error rates maintain the alpha value across all tests. Overall, we recommend the new tests for detecting location and scale changes under symmetric distributions when data are mixed designs of RCBD and CRD.

## Acknowledgement

The author thanks the anonymous referees for their comments and feedback on earlier version of this document.


## References


[1] D. Morrison, A test for equality of means of correlated variates with missing data on one response, Biometrika 61(1) (1973), 101-105.

[2] P. E. Lin and L. E. Stivers, On the difference of means with incomplete data, Biometrika 61(2) (1974), 325-334.

[3] G. Ekbohm, On comparing means in the paired case with incomplete data on both responses, Biometrika 63(2) (1976), 299-304.

[4] D. S. Bhoj, Testing equality of means of correlated variates with missing data on both responses, Biometrika 65(2) (1978), 225-228.

[5] S. Dubnicka, R. C. Blair and T. P. Hettmansperger, Rank-based procedures for mixed paired and two-sample designs, Journal of Modern Applied Statistical Methods 1(1) (2002), 32-41.

[6] R. C. Magel and R. Fu, Proposed nonparametric test for the mixed two-sample design, Journal of Statistical Theory and Practice 8(1) (2014), 221-237.

[7] R. Magel, J. Terpstra, K. Canonizado and J. I. Park, Nonparametric tests for mixed designs, Comm. Statist. Simulation Comput. 39(6) (2010), 1228-1250.

[8] S. Olet and R. Magel, A comparison of nonparametric tests in a mixed design for the simple tree alternative, International Journal for Research in Business Management and Accounting 3(3) (2017), 1-23.

[9] R. Magel and M. Ndungu, Nonparametric tests for ordering in completely randomized and randomized block mixed designs, Journal of Biometrics and Biostatistics 4(4) (2013), 20.

[10] Y. Lepage, A combination of Wilcoxon's and Ansari-Bradley's statistics, Biometrika 58(2) (1971), 213-217.

[11] M. Neuhauser and L. A. Hothorn, Parametric location-scale and scale trend tests based on Levene's transformation, Comput. Statist. Data Anal. 33(2) (2000), 189-200.

[12] M. Neuhauser, A.-K. Leuchs and D. Ball, A new location-scale test based on a combination of the ideas of Levene and Lepage, Biom. J. 53(3) (2011), 525-534.

[13] H. Levene, Robust tests for equality of variances, Contributions to Probability and Statistics, I. Olkin et al., eds., Stanford University Press, 1960, pp. 278-292.

[14] M. B. Brown and A. B. Forsythe, Robust tests for the equality of variances, J. Amer. Statist. Assoc. 69(346) (1974), 364-367.

[15] F. Khalawi and R. Magel, Proposed nonparametric tests for the simple tree alternative for location and scale testing in a mixed design, International Journal of Engineering Science Invention 11(60) (2022), 60-68.

[16] F. Khalawi and R. C. Magel, Location and scale testing in mixed design, International Journal of Statistics and Applied Mathematics 8(1) (2023), 88-94.

[17] SAS Institute Inc. SAS® 9.3 Foundation and CALL Routines: Reference, SAS Institute Inc., 2011.