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# ESTIMATION OF NEUTROSOPHIC POPULATION MEAN UTILIZING TWO AUXILIARY VARIABLES

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## Abstract

This article introduces a novel method for estimating the population mean by incorporating two auxiliary variables into the framework of the neutrosophic theory. We have proposed a generalized neutrosophic ratio type estimator for estimating the population mean utilizing two auxiliary information, and the efficiency of the proposed estimator was analyzed through an empirical study. The findings demonstrate that using information from two auxiliary variables improves performance compared to the Simple Random Sampling Without Replacement (SRSWOR) neutrosophic sample mean and standard neutrosophic ratio estimator using one auxiliary variable. Further, we have derived the condition where the proposed estimator performs better than the existing estimators. The process is evaluated using real

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life medical (neutrosophic) data, where contradictions and inconsistencies in medical information are commonly acknowledged, and the inclusion of indeterminacy is crucial. A simulation study has also been conducted to further validate and enhance the proposed approach.

## 1. Introduction

Statistical estimation aims to obtain estimators of the parameters of interest with greater precision. It is well established that incorporating accurate and reliable information into the estimation process enhances the efficiency and robustness of the estimators. In many practical settings, there arises a need to estimate the population mean or total, which can be effectively accomplished through the use of auxiliary information. Within the framework of classical statistics, several researchers have proposed different estimators for the finite population mean by incorporating auxiliary variables.

In survey sampling, the ratio estimation technique is a widely used method that enhances the accuracy of population mean estimation by utilizing auxiliary information. Instead of relying solely on the data collected for the study variable, this approach incorporates information from one or more auxiliary variables that are correlated with the study variable of interest. In many real-world applications, researchers often have access to prior knowledge or reliable information about one or two auxiliary variables can be leveraged to refine the estimation of the population mean for the variable under study. Olkin [9] pioneered a ratio estimator that utilized multiple auxiliary variables to improve estimation. Singh [14] suggested a chain ratio estimator with two auxiliary variables $X$ and $Z$. Abu-Dayyeh et al. [1] proposed two types of estimators for estimating the population mean of the variable of interest by utilizing two auxiliary variables. Kadilar and Cingi [6] derived a new estimator using two auxiliary variables. Shukla et al. [10] proposed families of estimators to estimate the population mean using the information on two different auxiliary variables in the SRSWOR scheme. Lu and Yan [7] proposed a class of ratio estimators of a finite population mean using two auxiliary variables. Muneer et al. [8] proposed a new class of estimators to estimate the finite population mean by using two auxiliary variables under two different sampling schemes, such as simple random sampling and stratified random sampling. Akingbade and Okafor [2] suggested a class of ratio type estimators with a linear combination using two auxiliary variables with some known population mean of the study variable. Singh and Nigam [11] suggested a generalized class of estimators for estimating the finite population mean of the study variable using information on two auxiliary variables.

Smarandache [15] proposed that neutrosophic statistics is an extension of classical and fuzzy statistical approaches designed to handle uncertainty, indeterminacy, and inconsistency in data. Unlike traditional statistics, which deals with precise values, neutrosophic statistics incorporates three components simultaneously: truth, indeterminacy, and falsity. This framework allows researchers to model situations where data may be incomplete, imprecise, or conflicting. As a result, neutrosophic statistics provides more flexibility and reliability in analyzing complex real-world problems where conventional methods may fall short.

The manuscript opens with an introduction to the study, followed by a brief review of the literature. Subsection 1.1 highlights the research gap that motivated this work. Section 2 presents the notations and terminology used throughout the article. Section 3 introduces the proposed generalized estimator, along with the derivation of its bias and Mean Squared Error (MSE). Subsections 3.1 and 3.2 further discuss the remarks and efficiency of the proposed estimator. Sections 4 and 5 are devoted to numerical study and the simulation study, respectively. Finally, Section 6 provides the conclusion and outlines the limitations of the study.

### 1.1. Research gap

Tahir et al. [17] were the first to explore the problem of estimating the population mean under neutrosophic data by introducing ratio type estimators that utilized a single auxiliary variable. Building on this, Tahir et al. [16] proposed a generalized Neutrosophic Ratio Type Exponential Estimator (NRTEE) for the estimation of location parameters, with particular focus on the population mean. Subsequently, Alqudah et al. [3] developed a generalized estimator for the neutrosophic ratio-product approach based on the ordinary least squares method. More recently, Singh and Gupta [12] advanced the field further by formulating a ratio-cum-product exponential estimator that incorporated two auxiliary variables, with applications demonstrated in agricultural datasets. Inspired by these developments, the present study introduces a generalized neutrosophic ratio estimator using two auxiliary variables aimed at improving the estimation of the population mean.

## 2. Notation and Terminology

Suppose we have a neutrosophic random sample of size $n_N$ drawn from a finite neutrosophic population of size $N \in [N_L, N_U]$ units denoted by $(T_1, T_2, ..., T_N)$. Let $y_N(i)$ be the $i^{\text{th}}$ sample observation of our neutrosophic data, which is of the form $y_N(i) \in [y_L, y_U]$ and similarly, for auxiliary variables, we have $x_N(i) \in [x_L, x_U]$ and $z_N(i) \in [z_L, z_U]$ which are correlated to our variable of interest *Y*. Let $\bar{Y}_N \in [\bar{Y}_L, \bar{Y}_U]$ be the population mean of the neutrosophic variable of interest, and $\bar{X}_N \in [\bar{X}_L, \bar{X}_U]$ and $\bar{Z}_N \in [\bar{Z}_L, \bar{Z}_U]$ are the population means of auxiliary neutrosophic variables *X* and *Z*. Let $\bar{y}_N(i) \in [\bar{y}_L, \bar{y}_U]$ be the mean of the neutrosophic sample of the variable of interest. $\bar{x}_N(i) \in [\bar{x}_L, \bar{x}_U]$ and $\bar{z}_N(i) \in [\bar{z}_L, \bar{z}_U]$ are the means of the neutrosophic sample of the variables *X* and *Z*. $S^2_{yN} \in [S^2_{yNL}, S^2_{yNU}]$, $S^2_{xN} \in [S^2_{xNL}, S^2_{xNU}]$ and $S^2_{zN} \in [S^2_{zNL}, S^2_{zNU}]$ are the variances of the neutrosophic variable *Y* and auxiliary variables *X* and *Z*. $C_{yN} \in [C_{yNL}, C_{yNU}]$, $C_{xN} \in [C_{xNL}, C_{xNU}]$ and $C_{zN} \in [C_{zNL}, C_{zNU}]$ are the coefficients of variation of the neutrosophic variable *Y* and auxiliary variables *X* and *Z*. $\rho_{N_{yx}} \in [\rho_{N_{yxL}}, \rho_{N_{yxU}}]$ is the correlation between neutrosophic variables $Y$ and $X$. $\rho_{N_{yz}} \in [\rho_{N_{yzL}}, \rho_{N_{yzU}}]$ and $\rho_{N_{xz}} \in [\rho_{N_{xzL}}, \rho_{N_{xzU}}]$ are the correlations between neutrosophic variables $Y$ and $Z$ and the auxiliary variables $X$ and $Z$, respectively.

$f = n_N / N$ is the sampling fraction,

$\theta_N = \dfrac{1-f}{n_N}$ is the finite population correction.

In SRSWOR, the estimator of the neutrosophic population mean $\overline{Y}_N$ is $\bar{y}_N$ and its variance is

$$V(\bar{y}_N) = \frac{(1-f)}{n_N} S_{yN}^2 \quad \text{or} \quad \theta_N \bar{y}_N^2 C_{yN}^2. \tag{1}$$

Tahir et al. [17] introduced a neutrosophic ratio estimator to estimate the finite population mean using an auxiliary variable $X$,

$$\bar{y}_{RNx} = \frac{\bar{y}_N}{\bar{x}_N} \overline{X}_N. \tag{2}$$

The bias and MSE of $\hat{\bar{y}}_{RNx}$ up to first-order approximation are given by

$$Bias(\hat{\bar{y}}_{RNx}) = \theta_N \overline{Y}_N [C_{xN}^2 - C_{xN} C_{yN} \rho_{xyN}], \tag{3}$$

$$MSE(\hat{\bar{y}}_{RNx}) = \theta_N \overline{Y}_N^2 [C_{yN}^2 + C_{xN}^2 - 2C_{xN} C_{yN} \rho_{xyN}]. \tag{4}$$

Further, one more neutrosophic ratio estimator to estimate the finite population mean using another auxiliary variable $Z$ is as follows:

$$\bar{y}_{RNz} = \frac{\bar{y}_N}{\overline{Z}_N} \overline{Z}_N. \tag{5}$$

The bias and MSE of $\hat{\bar{y}}_{RNz}$ up to first-order approximation are given by

$$Bias(\hat{\bar{y}}_{RNz}) = \theta_N \overline{Y}_N [C_{zN}^2 - C_{zN} C_{yN} \rho_{zyN}], \tag{6}$$

$$MSE(\hat{\bar{y}}_{RNz}) = \theta_N \bar{Y}_N^2 [C_{yN}^2 + C_{zN}^2 - 2C_{zN}C_{yN}\rho_{zyN}], \quad (7)$$

where $\theta_N \in [\theta_L, \theta_U]$ and $n_N \in [n_L, n_U]$.

## 3. The Proposed Generalized Estimator

Following Tahir et al. [17] and Singh and Gupta [12], making use of two auxiliary variables, the following neutrosophic multivariate estimator is proposed:

$$\hat{\bar{y}}_{NR} = \bar{Y}_N \frac{\bar{X}_N}{\bar{x}_N} \frac{\bar{Z}_N}{\bar{z}_N}. \quad (8)$$

Mean squared error of the proposed estimator is given as follows:

$$MSE(\hat{\bar{y}}_{NR}) = \frac{(1-f)}{n_N} \bar{Y}^2 (C_{yN}^2 + C_{xN}^2 + C_{zN}^2 - 2\rho_{Nyx}C_{yN}C_{xN} - 2\rho_{Nyz}C_{yN}C_{zN} + 2\rho_{Nxz}C_{xN}C_{zN}). \quad (9)$$

Further, Singh and Gupta [12] proposed a generalized neutrosophic ratio estimator using two auxiliary variables:

$$\bar{y}_{NMr} = \bar{Y}_N \left(\frac{\bar{X}_N}{\bar{x}_N}\right)^{\alpha_{1N}} \left(\frac{\bar{Z}_N}{\bar{z}_N}\right)^{\alpha_{2N}}, \quad (10)$$

where $\alpha_{1N} \in (\alpha_{1L}, \alpha_{1U})$ and $\alpha_{2N} \in (\alpha_{2L}, \alpha_{2U})$.

Next, we derive the bias and Mean Squared Error (MSE) of the proposed estimator.

Let us define

$$e_0 = \frac{\bar{y}_N - \bar{Y}_N}{\bar{Y}_N}, \quad e_1 = \frac{\bar{x}_N - \bar{X}_N}{\bar{X}_N}, \quad e_2 = \frac{\bar{z}_N - \bar{Z}_N}{\bar{Z}_N},$$

$$E(e_0) = E(e_1) = E(e_2) = 0,$$

$$E(e_0^2) = \frac{1-f}{n_N} C_{yN}^2, \quad E(e_1^2) = \frac{1-f}{n_N} C_{xN}^2, \quad E(e_2^2) = \frac{1-f}{n_N} C_{zN}^2,$$

$$E(e_0 e_1) = \frac{f}{n_N} \rho_{Nyx} C_{yN} C_{xN}, \quad E(e_0 e_2) = \frac{f}{n_N} \rho_{Nyz} C_{yN} C_{zN},$$

$$E(e_1 e_2) = \frac{f}{n_N} \rho_{Nxz} C_{xN} C_{zN},$$

$$\begin{aligned}\bar{y}_{NMr} &= \bar{Y}_N (1 + e_0)(1 + e_1)^{-\alpha_{1N}} (1 + e_2)^{-\alpha_{2N}} \\ &= \bar{Y}_N \Big[ 1 + e_0 - \alpha_{1N} e_1 - \alpha_{2N} e_2 - \alpha_{1N} e_0 e_1 \\ &\qquad - \alpha_{2N} e_0 e_2 + \alpha_{1N} \alpha_{2N} e_1 e_2 \\ &\qquad + \frac{\alpha_{1N}(\alpha_{1N} + 1)}{2} e_1^2 + \frac{\alpha_{2N}(\alpha_{2N} + 1)}{2} e_2^2 + \cdots \Big],\end{aligned}$$

$$\begin{aligned}E(\bar{y}_{NMr}) = \bar{Y}_N - \bar{Y}_N \Big[ &\alpha_{1N} \frac{f}{n_N} \rho_{Nyx} C_{yN} C_{xN} + \alpha_{2N} \frac{f}{n_N} \rho_{Nyz} C_{yN} C_{zN} \\ &- \alpha_{1N} \alpha_{2N} \frac{f}{n_N} \rho_{Nxz} C_{xN} C_{zN} \\ &- \frac{\alpha_{1N}(\alpha_{1N} + 1)}{2} \frac{f}{n_N} C_{xN}^2 + \frac{\alpha_{2N}(\alpha_{2N} + 1)}{2} \frac{f}{n_N} C_{zN}^2 \Big],\end{aligned}$$

$$\begin{aligned}Bias(\bar{y}_{NMr}) = \bar{Y}_N \frac{1-f}{n_N} \Big[ &\alpha_{1N} \alpha_{2N} \rho_{Nxz} C_{xN} C_{zN} \\ &- \alpha_{1N} \rho_{Nyx} C_{yN} C_{xN} - \alpha_{2N} \rho_{Nyz} C_{yN} C_{zN} \\ &+ \frac{\alpha_{1N}(\alpha_{1N} + 1)}{2} C_{xN}^2 + \frac{\alpha_{2N}(\alpha_{2N} + 1)}{2} C_{zN}^2 \Big], \quad (11)\end{aligned}$$

$$MSE(\bar{y}_{NMr}) = E(\bar{y}_{NMr} - \bar{Y}_N)^2$$

$$\cong \bar{Y}_N^2 \frac{1-f}{n_N}[C_{yN}^2 + \alpha_{1N}^2 C_{xN}^2 + \alpha_{2N}^2 C_{zN}^2$$

$$- 2\alpha_{1N}\rho_{Nyx}C_{yN}C_{xN} - 2\alpha_{2N}\rho_{Nyz}C_{yN}C_{zN}$$

$$+ 2\alpha_{1N}\alpha_{2N}\rho_{Nxz}C_{xN}C_{zN}]. \quad (12)$$

The bias and mean square error of the proposed estimator are given below:

$$Bias(\bar{y}_{NMr}) = \bar{Y}_N \frac{1-f}{n_N}[\alpha_{1N}\alpha_{2N}\rho_{Nxz}C_{xN}C_{zN}$$

$$- \alpha_{1N}\rho_{Nyx}C_{yN}C_{xN} - \alpha_{2N}\rho_{Nyz}C_{yN}C_{zN}$$

$$+ \frac{\alpha_{1N}(\alpha_{1N}+1)}{2}C_{xN}^2 + \frac{\alpha_{2N}(\alpha_{2N}+1)}{2}C_{zN}^2],$$

$$MSE(\bar{y}_{NMr}) \cong \bar{Y}_N^2 \frac{1-f}{n_N}[C_{yN}^2 + \alpha_{1N}^2 C_{xN}^2$$

$$+ \alpha_{2N}^2 C_{zN}^2 - 2\alpha_{1N}\rho_{Nyx}C_{yN}C_{xN}$$

$$- 2\alpha_{2N}\rho_{Nyz}C_{yN}C_{zN} + 2\alpha_{1N}\alpha_{2N}\rho_{Nxz}C_{xN}C_{zN}].$$

Now, we determine the optimal values of $\alpha_N$ that ensures the robustness of the proposed estimator:

$$\frac{\partial MSE(\bar{y}_{NMr})}{\partial \alpha_{1N}} = \bar{Y}_N^2 \frac{f}{n_N}[2\alpha_{1N}C_{xN}^2 + 2\rho_{Nyx}C_{yN}C_{xN} + 2\alpha_{2N}\rho_{Nxz}C_{xN}C_{zN}]$$

$$\frac{\partial MSE(\bar{y}_{NMr})}{\partial \alpha_{1N}} = \bar{Y}_N^2 \frac{f}{n_N} 2[\alpha_{1N}C_{xN}^2 + \rho_{Nyx}C_{yN}C_{xN} + \alpha_{2N}\rho_{Nxz}C_{xN}C_{zN}]$$

$$\frac{\partial MSE(\bar{y}_{NMr})}{\partial \alpha_{1N}} = 0 \Rightarrow \alpha_{1N}C_{xN}^2 + \rho_{Nyx}C_{yN}C_{xN} + \alpha_{2N}\rho_{Nxz}C_{xN}C_{zN} = 0, \quad (13)$$

$$\frac{\partial MSE(\bar{y}_{NMr})}{\partial \alpha_{2N}} = 0 \Rightarrow \alpha_{2N}C_{zN}^2 + \rho_{Nyz}C_{yN}C_{zN} + \alpha_{1N}\rho_{Nxz}C_{xN}C_{zN} = 0. \quad (14)$$

Solving equations (13) and (14), we get the optimum values of $\alpha_{1N}$ and $\alpha_{2N}$ as follows:

$$\alpha_{1N} = \frac{C_{yN}(\rho_{Nyx} - \rho_{Nyz}\rho_{Nxz})}{C_{xN}(1 - \rho_{Nxz}^2)}, \quad (15)$$

$$\alpha_{2N} = \frac{C_{yN}(\rho_{Nyx} - \rho_{Nyx}\rho_{Nxz})}{C_{zN}(1 - \rho_{Nxz}^2)}. \quad (16)$$

### 3.1. Remarks

**Remark 1.** Neutrosophic ratio estimator with a single auxiliary variable

**Case 1.** When $\alpha_{1N} = (0, 0)$ and $\alpha_{2N} = (1, 1)$ provide a neutrosophic ratio estimator $\bar{y}_{RNz}$ with the single auxiliary variable, *Z*. This corresponds to a scenario where the emphasis is on *Z*, while *X* is excluded from the estimation.

**Case 2.** When $\alpha_{1N} = (1, 1)$ and $\alpha_{2N} = (0, 0)$ give a neutrosophic ratio estimator $\bar{y}_{RNx}$ with the single auxiliary variable *X*. In this case, only *X* is utilized for estimation, with no contribution from *Z*.

**Remark 2.** Neutrosophic multivariate ratio estimator with two auxiliary variables

If $\alpha_{1N} = \alpha_{2N} = (1, 1)$, then both auxiliary variables *X* and *Z* contribute positively. This combination results in the neutrosophic multivariate ratio estimator $\hat{\bar{y}}_{NMr}$, where both variables are utilized in harmony to improve estimation efficiency.

**Remark 3.** Neutrosophic multivariate product estimator

If $\alpha_{1N} = \alpha_{2N} = (-1, -1)$, then both auxiliary variables *X* and *Z* contribute with a negative relationship (inverted ratio). This leads to the neutrosophic multivariate product estimator, which is suited for scenarios where the negative relationship is more appropriate than the positive relationship for estimation.

### 3.2. Efficiency comparison

Let us obtain the condition under which the proposed estimator performs better than the SRSWOR neutrosophic sample mean. From equations (1) and (12), we get the following expression:

$$MSE(\bar{y}_{NMr}) \leq V(\bar{y}_N)$$

if

$$\alpha_{1N}^2 C_{xN}^2 + \alpha_2^2 C_{zN}^2 \leq 2(\alpha_{1N}\rho_{Nyx}C_{yN}C_{xN} + \alpha_2\rho_{Nyz}C_{yN}C_{zN} - \alpha_{1N}\alpha_{2N}\rho_{Nxz}C_{xN}C_{zN}). \quad (17)$$

From the above equations (4) and (12), we derive the efficiency condition of the proposed estimator to the neutrosophic ratio estimator using a single auxiliary variable:

$$MSE(\bar{y}_{NMr}) \leq MSE(\hat{\bar{y}}_{RNx})$$

if

$$(\alpha_{1N}^2 - 1)C_{xN}^2 + \alpha_{2N}^2 C_{zN}^2 - 2\rho_{Nyx}C_{yN}C_{xN}(\alpha_{1N} - 1) \leq 2(\alpha_{2N}\rho_{Nyz}C_{yN}C_{zN} - \alpha_{2N}\rho_{Nxz}C_{xN}C_{zN}). \quad (18)$$

Furthermore, for the proposed estimators (*p*) to demonstrate their efficiency regarded as superior to existing estimators (*e*), they must attain the minimum possible Mean Squared Error (MSE).

The Percent Relative Efficiencies (PREs) of the proposed estimators compared to the existing estimator (*e*) are calculated using the following formula:

$$PRE(p) = \frac{MSE(e)}{MSE(p)} * 100.$$

If $PRE(p) > 100,$ then the proposed estimators outperform the existing estimators. Additionally, an increasing PRE signifies that the proposed estimators are becoming more efficient.

## 4. Numerical Evaluation of Neutrosophic Data

Neutrosophic observations can take various forms, with neutrosophic numbers potentially defined within an unknown interval $[a, b]$. In this framework, neutrosophic values are represented as $Z_N = Z_L + Z_U I_N$, where $I_N \in [I_L, I_U], Z_N \in [Z_L, Z_U]$. The notation $N$ is used to signify a neutrosophic number. For the numerical study, we have considered a real-life indeterminacy interval data of Chronic Kidney Disease (CKD). Medical professionals prioritize making precise and accurate decisions to safeguard human health, but managing uncertainty and indeterminacy poses significant challenges in this domain. Neutrosophic statistics holds a vital role in the healthcare sector, aiding in the analysis of neutrosophic data within the medical field [5, Chapter 19]). We have taken a Chronic Kidney Disease (CKD) dataset from the UCI repository, publicly published

(https://archive.ics.uci.edu/dataset/857/risk+factor+prediction+of+chronic+kidney+disease).

Accordingly, our neutrosophic observations are confined to the interval $Z_N \in [a, b]$, with $a$ and $b$ indicating the lower and upper bounds of the neutrosophic data, respectively. In our dataset, GFR test results, measured at intervals to monitor kidney function, were treated as the independent variable. The population parameters of the neutrosophic dataset are displayed in the table below:

**Table 1.** Parameters and constants of the neutrosophic population

| $N$ | $n$ | $\bar{y}_N$ | $\bar{x}_N$ |
|---|---|---|---|
| (155, 155) | (45, 45) | (52.23, 72.28) | (134.91, 139.90) |
| $\bar{z}_N$ | $S^2_{yN}$ | $S^2_{xN}$ | $S^2_{zN}$ |
| (34.91, 38.1) | (2438.42, 2834.47) | (32.88, 32.88) | (47.78, 47.78) |

| $C_{yN}$ | $C_{xN}$ | $C_{zN}$ | $\rho_{N_{yx}}$ |
|---|---|---|---|
| (94.36, 73.42) | (4.25, 4.09) | (19.80, 17.81) | (0.43, 0.44) |
| $\rho_{N_{yz}}$ | $\rho_{N_{xz}}$ | | |
| (0.49, 0.51) | (0.45, 0.45) | | |

The table below presents the mean square error $MSE_{NR} \in (MSE_L, MSE_U)$ and it demonstrates that the proposed estimator achieved the lowest MSE at the optimum values of $\alpha_{1N}$ and $\alpha_{2N}$.

**Table 2.** MSE of the numerical data for varying values of $\alpha_{1N}$ and $\alpha_{2N}$

| $\alpha_{1N}$ | $\alpha_{2N}$ | **Estimator** | $MSE_{NR}$ |
|---|---|---|---|
| (0, 0) | (0, 0) | $\bar{y}$ | (385257.6, 447014.0) |
| (1, 1) | (0, 0) | $\bar{y}_{RNx}$ | (371308.6, 426537.5) |
| (0, 0) | (1, 1) | $\bar{y}_{RNz}$ | (322001.4, 362276.9) |
| (1, 1) | (1, 1) | $\hat{\bar{y}}_{NR}$ | (**311311.6, 347245.2**) |
| Optimum value (5.65, 4.685) | Optimum value (1.837, 1.633) | $\hat{\bar{y}}_{NMr}$ | **(270168.6, 305274.1)** |

The table below presents the percent relative efficiency of the proposed estimators:

**Table 3.** Percent relative efficiency of the proposed estimator

| **Estimator** | **PREs** |
|---|---|
| $\bar{y}$ | (142.59, 146.43) |
| $\bar{y}_{RNx}$ | (137.44, 139.72) |
| $\bar{y}_{RNz}$ | (119.19, 118.67) |
| $\hat{\bar{y}}_{NR}$ | (115.23, 113.75) |

From Table 3, the percent relative efficiency of the proposed estimator ranges from (115.23, 113.75) to (142.59, 146.43). This implies that the proposed estimator achieves superior performance compared to the existing estimators.

## 5. Simulation Study

We simulated 1000 normal random variates from a multivariate normal distribution, where

$$(Y_{NL}, X_{NL}, Z_{NL}) \sim [(\mu_{yL}, \sigma^2_{yL}), (\mu_{xL}, \sigma^2_{xL}), (\mu_{zL}, \sigma^2_{zL})]$$

and

$$(Y_{NU}, X_{NU}, Z_{NU}) \sim [(\mu_{yU}, \sigma^2_{yU}), (\mu_{xU}, \sigma^2_{xU}), (\mu_{zU}, \sigma^2_{zU})].$$

The correlation between the variables is 0.9 with the covariance matrix Singh et al. [13]:

$$\Sigma_L = \begin{bmatrix} \sigma^2_{xL} & \rho_{xyL}\sigma_{xL}\sigma_{yL} & \rho_{xzL}\sigma_{xL}\sigma_{zL} \\ \rho_{xyL}\sigma_{xL}\sigma_{yL} & \sigma^2_{yL} & \rho_{yzL}\sigma_{yL}\sigma_{zL} \\ \rho_{xzL}\sigma_{xL}\sigma_{zL} & \rho_{yzL}\sigma_{yL}\sigma_{zL} & \sigma^2_{zL} \end{bmatrix} \text{ for lower bounds,}$$

$$\Sigma_U = \begin{bmatrix} \sigma^2_{xU} & \rho_{xyU}\sigma_{xLU}\sigma_{yU} & \rho_{xzU}\sigma_{xU}\sigma_{zU} \\ \rho_{xyU}\sigma_{xU}\sigma_{yU} & \sigma^2_{yU} & \rho_{yzU}\sigma_{yU}\sigma_{zU} \\ \rho_{xzU}\sigma_{xU}\sigma_{zU} & \rho_{yzU}\sigma_{yU}\sigma_{zU} & \sigma^2_{zU} \end{bmatrix} \text{ for upper bounds.}$$

Consider SRSWOR for sample size of 540. The following table shows the mean squared error of the simulated data:

**Table 4.** Mean squared error of the simulated data for varying $\alpha_{1N}$ and $\alpha_{2N}$

| $\alpha_{1N}$ | $\alpha_{2N}$ | **Simulation study** |
|---|---|---|
| (0, 0) | (0, 0) | (20431.22, 23790.75) |
| (1, 1) | (0, 0) | (13685.18, 14993.51) |
| (0, 0) | (1, 1) | (18830.42, 20879.64) |
| (1, 1) | (1, 1) | **(12429.8, 13283.78)** |
| Optimum value (2.11, 1.85) | Optimum value (11.06, 9.02) | **(3246.74, 3782.39)** |

The table below presents the percent relative efficiency of the proposed estimator for the simulated data.

**Table 5.** Percent relative efficiency for the simulated data

| Estimator | (PREs) |
|---|---|
| $\bar{y}$ | (629.284, 628.987) |
| $\bar{y}_{RNx}$ | (421.505, 396.403) |
| $\bar{y}_{RNz}$ | (579.979, 552.022) |
| $\hat{\bar{y}}_{NR}$ | (382.834, 351.200) |

From Table 5, the percent relative efficiency of the proposed estimator ranges from (382.834, 351.200) to (629.284, 628.987). This demonstrates that the proposed generalized neutrosophic ratio estimator outperforms the SRSWOR sample mean, the conventional ratio estimator with a single auxiliary variable, and the proposed neutrosophic ratio estimator that incorporates information from two auxiliary variables.

## 6. Conclusion

In this study, we introduced a generalized neutrosophic ratio estimator for the estimation of the population mean by incorporating two auxiliary variables. The expressions for the bias and Mean Squared Error (MSE) of the proposed estimator are derived, and the optimum conditions under which it outperforms existing estimators are established. The numerical evaluation demonstrates that the proposed estimator consistently yields a lower MSE compared to traditional counterparts. These theoretical results are further reinforced through an empirical application using real data and validated by means of a simulation study.

This article employs medical data in the numerical evaluation; a researcher may conduct empirical investigations across various domains whenever the estimation of the population mean is relevant. Although the proposed estimator is formulated as a generalized and robust approach, one notable limitation lies in its non-uniqueness. The non-uniqueness provides researchers with flexibility in adapting the estimator to different sampling situations, datasets, or auxiliary information structures. This adaptability enhances the practical utility of the estimator, making it applicable across a wide range of real-world problems where data characteristics and auxiliary information may vary.

**Future study**

The present study introduces a generalized neutrosophic estimator that incorporates two auxiliary variables under the framework of Simple Random Sampling Without Replacement (SRSWOR). A natural extension of this work would be to explore its applicability within alternative sampling designs, such as stratified sampling and ranked set sampling.

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