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AVERAGE TOTAL DOMINATION ON A PYTHAGOREAN ANTI FUZZY GRAPH

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Abstract

In this paper, we introduce the concept of average total domination on a Pythagorean anti fuzzy graph and determine the exact value of average total domination number of some standard Pythagorean anti fuzzy graphs.

1. Introduction

The study of dominating set in graphs was begun by Ore and Berge. Rosenfeld introduced the notion of fuzzy graph in 1975. In fuzzy graph, we attain only minimum value among objects. Sometimes, we need to attain maximum values. In such cases, the problem is constructed as anti fuzzy graph model. In 2016, the concept of anti fuzzy graph was introduced by Seethalakshmi and Gnanajothi [5]. Later, Muthuraj and Sasirekha [4] extended this concept and also introduced the idea of domination on anti fuzzy graph in 2018. Maheswari et al. [1] defined the notion of a Pythagorean anti fuzzy graph model. In this paper, we introduce the concept of average total domination on a Pythagorean anti fuzzy graph with suitable example. We determine the exact value of average total domination number on some standard Pythagorean anti fuzzy graph. Further, its properties are investigated.

2. Preliminaries

Definition 2.1 [5]**.** An *anti fuzzy graph* $\mathcal{A} = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ with $\mu(a, b) \geq \sigma(a) \vee \sigma(b)$ for all a, b in V, where V is a finite nonempty set and $\vee$ denotes maximum.

Definition 2.2 [5]**.** An anti fuzzy graph $\mathcal{A} = (\sigma, \mu)$ is said to be *complete* if the underlying graph $\mathcal{A}^*$ is complete and $\mu(a, b) = \sigma(a) \vee \sigma(b)$ for all (a, b) in E.

Definition 2.3 [1]**.** A *Pythagorean anti fuzzy set* on a universe χ is an object of the form $\mathcal{P} = \{\langle a, \mu_{\mathcal{P}}(a), \nu_{\mathcal{P}}(a)\rangle / a \in \chi\}$, where $\mu_{\mathcal{P}} : \chi \to [0, 1]$

and $\nu_{\mathcal{P}} : \chi \to [0, 1]$ represent the membership and non-membership grade of a and satisfies the condition $0 \le \mu_{\mathcal{P}}^2(a) + \nu_{\mathcal{P}}^2(a) \le 1$ for all $a \in \chi$.

Definition 2.4 [1]**.** A Pythagorean anti fuzzy set on $\chi \times \chi$ is said to be a *Pythagorean anti fuzzy relation* on χ, denoted by

$$\mathcal{Q} = \{\langle ab, \mu_{\mathcal{Q}}(ab), \nu_{\mathcal{Q}}(ab)\rangle / ab \in \chi \times \chi\},$$

where $\mu_{\mathcal{Q}} : \chi \times \chi \to [0, 1]$ and $\nu_{\mathcal{Q}} : \chi \times \chi \to [0, 1]$ represent the membership and non-membership grade of $\mathcal{Q}$ and satisfies the condition $0 \le \mu_{\mathcal{Q}}^2(ab) + \nu_{\mathcal{Q}}^2(ab) \le 1$ for all $a, b \in \chi$.

Definition 2.5 [1]**.** A *Pythagorean anti fuzzy graph* on a nonempty set χ is a pair $\mathcal{A}^{**} = (\mathcal{P}, \mathcal{Q})$ with $\mathcal{P}$ a Pythagorean anti fuzzy set on χ and $\mathcal{Q}$ a Pythagorean anti fuzzy relation on χ such that

$$\mu_{\mathcal{Q}}(ab) \ge \mu_{\mathcal{P}}(a) \vee \mu_{\mathcal{P}}(b)$$

and

$$\nu_{\mathcal{Q}}(ab) \le \nu_{\mathcal{P}}(a) \wedge \nu_{\mathcal{P}}(b)$$

for all $a, b \in \chi$, where $\mu_{\mathcal{Q}} : \chi \times \chi \to [0, 1]$ and $\nu_{\mathcal{Q}} : \chi \times \chi \to [0, 1]$ represent the membership and non-membership grade of $\mathcal{Q}$, respectively.

3. Main Results

Definition 3.1. Let $\mathcal{A}^{**}$ be a Pythagorean anti fuzzy graph. The minimal dominating set of $\mathcal{A}^{**}$ that contains the vertex a and the subgraph induced by *D* has no isolated vertices is called the *lower total dominating set*. Among all lower total dominating sets in $\mathcal{A}^{**}$, the maximum Pythagorean fuzzy cardinality is called the *lower total domination number* on a Pythagorean anti fuzzy graph. It is denoted by $\gamma_{at}(\mathcal{A}^{**})$. The dominating set of $\gamma_{at}(\mathcal{A}^{**})$

is the γ_{at} -set of a Pythagorean anti fuzzy graph. The γ_{at} -set is denoted by D_{at}.

Definition 3.2. The *average total domination number* on a Pythagorean anti fuzzy graph is defined as $\frac{\sum_{a\in V}\gamma_{at}(\mathcal{A}^{**})}{|V|}$, where $\gamma_{at}(\mathcal{A}^{**})$ is the lower total domination number. It is denoted by $\gamma_{Agt}(\mathcal{A}^{**})$.

Example 3.3.

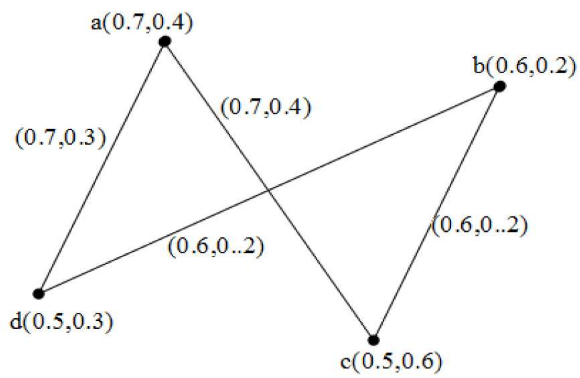

Figure 1. Pythagorean anti fuzzy graph.

Table 1. Lower total domination number on a Pythagorean anti fuzzy graph

$a \in V(\mathcal{A}^{**})$	γ_{at} -set	$\gamma_{at}(\mathcal{A}^{**})$
a	{a, d}	1.25
b	{b, d}	1.30
c	{c, b}	1.15
d	{d, b}	1.30

$$\gamma_{Agt}(\mathcal{A}^{**}) = \frac{\sum_{a\in V}\gamma_{at}(\mathcal{A}^{**})}{|V|} = \frac{1.25+1.3+1.15+1.3}{4} = 1.25.$$

Theorem 3.4. *If* $\mathcal{A}^{**}$ *is the Pythagorean anti fuzzy path with equal Pythagorean fuzzy vertex and edge cardinality*, *then*

$$\gamma_{Ag}(\mathcal{A}^{**}) = \begin{cases} \dfrac{(8m^2 - m - 1)|\,a\,|}{|\,V\,|} & for\ n = 4m - 1, \\ \dfrac{(4mn + n)|\,a\,|}{2|\,V\,|} & for\ n = 4m, \\ \dfrac{(8m^2 + 7m + 2)|\,a\,|}{|\,V\,|} & for\ n = 4m + 1, \\ \dfrac{2(m+1)|\,a\,|}{|\,V\,|} & for\ n = 4m + 2, \end{cases}$$

where $m \geq 1, |\,V\,| = n.$

Proof. Let $\mathcal{A}^{**}$ be the Pythagorean anti fuzzy path with equal Pythagorean fuzzy vertex and edge cardinality and the vertex set be $\{a_1, a_2, ..., a_n\}$. Then a_1 is adjacent to a_2 and a_{n-1} is adjacent to a_n. a_i is adjacent to a_{i-1} and $a_{i+1}, 2 \leq i \leq n-1.$ It is obvious that a_1 dominates a_2 and a_{n-1} dominates a_n. a_i is dominated by a_{i-1} and $a_{i+1}, 2 \leq i \leq n-1.$

Let D_{at} be the Pythagorean anti fuzzy lower total dominating set.

Case 1. For $n = 4m - 1$

The set $\sum_{a \in V} D_{at}$ having Pythagorean fuzzy cardinality $2m|\,a\,|$ in $3m$ times and $(2m+1)|\,a\,|$ in $m - 1$ times.

Therefore,

$$\sum_{a \in V} \gamma_{at}(\mathcal{A}^{**}) = [(2m)(3m) + (2m+1)(m-1)]|\,a\,|$$

$$= [6m^2 + 2m^2 - 2m + m - 1]|\,a\,|$$

$$= [8m^2 - m - 1]|\,a\,|.$$

Hence, $\gamma_{Agt}(\mathcal{A}^{**}) = \dfrac{(8m^2 - m - 1)|\,a\,|}{|\,V\,|}.$

Case 2. For $n = 4m$

The set $\sum_{a \in V} D_{at}$ having Pythagorean fuzzy cardinality $(2m+1)|a|$ in $\frac{n}{2}$ times and $(2m)|a|$ in $\frac{n}{2}$ times.

Therefore,

$$\sum_{a \in V} \gamma_{at}(\mathcal{A}^{**}) = \left[(2m+1)\frac{n}{2} + (2m)\frac{n}{2}\right]|a|$$

$$= \left[\frac{2mn + n + 2mn}{2}\right]|a|$$

$$= \left[\frac{4mn + n}{2}\right]|a|.$$

Hence, $\gamma_{Agt}(\mathcal{A}^{**}) = \frac{(4mn+n)|a|}{2|V|}$.

Case 3. For $n = 4m+1$

The set $\sum_{a \in V} D_{at}$ having Pythagorean fuzzy cardinality $(2m+2)|a|$ in $m+1$ times and $(2m+1)|a|$ in $3m$ times.

Therefore,

$$\sum_{a \in V} \gamma_{at}(\mathcal{A}^{**}) = [2(m+1)(m+1) + (2m+1)(3m)]|a|$$

$$= [2(m+1)^2 + 6m^2 + 3m]|a|$$

$$= [8m^2 + 7m + 2]|a|.$$

Hence, $\gamma_{Agt}(\mathcal{A}^{**}) = \frac{(8m^2 + 7m + 2)|a|}{|V|}$.

Case 4. For $n = 4m+2$

The set $\sum_{a \in V} D_{at}$ has Pythagorean fuzzy cardinality $|a|$ in $2m+2$ times.

Therefore,

$$\sum_{a \in V} \gamma_{at}(\mathcal{A}^{**}) = (2m + 2)|\, a\,| = 2(m + 1)|\, a\,|.$$

Hence, $\gamma_{Agt}(\mathcal{A}^{**}) = \dfrac{2(m+1)|\, a\,|}{|V|}.$

Theorem 3.5. *If* $\mathcal{A}^{**} = (\mathcal{P}, \mathcal{Q})$ *is a complete Pythagorean anti fuzzy graph* K_n, *then* $\gamma_{Agt}(K_n) = \dfrac{\sum_{i=1}^{n}(|\, a_i\,| + |\, a_j\,|)}{|V|}$, *where* a_j *is the vertex of maximum Pythagorean fuzzy cardinality among all vertices other than* a_i, $i = 1, 2, ..., n$.

Proof. Let K_n be the complete Pythagorean anti fuzzy graph and the vertex set be $\{a_1, a_2, ..., a_{n-1}, a_n\}$. Then by the definition of complete Pythagorean anti fuzzy complete graph, each a_i dominates all other Pythagorean fuzzy vertices, $i = 1, 2, ..., n$.

Let D_{at} be the Pythagorean anti fuzzy lower total dominating set. Then, $D_{a_i t} = \{a_i, a_j\}$, where a_j is the vertex of maximum Pythagorean fuzzy cardinality among all vertices other than a_i, $i = 1, 2, ..., n$.

Therefore,

$$\gamma_{a_1} = |\, a_1\,| + \max\{|\, a_2\,|, |\, a_3\,|, ..., |\, a_n\,|\},$$

$$\gamma_{a_2} = |\, a_2\,| + \max\{|\, a_1\,|, |\, a_3\,|, ..., |\, a_n\,|\}, ...,$$

$$\gamma_{a_n} = |\, a_n\,| + \max\{|\, a_1\,|, |\, a_2\,|, ..., |\, a_{n-1}\,|\},$$

$$\begin{aligned}\sum_{a \in V} \gamma_{at}(\mathcal{A}^{**}) &= \gamma_{a_1} + \gamma_{a_2} + \cdots + \gamma_{a_n} \\ &= |\, a_1\,| + \max\{|\, a_2\,|, |\, a_3\,|, ..., |\, a_n\,|\} \\ &\quad + |\, a_2\,| + \max\{|\, a_1\,|, |\, a_3\,|, ..., |\, a_n\,|\}\end{aligned}$$

$$+ \cdots + |a_n| + \max\{|a_1|, |a_2|, ..., |a_{n-1}|\}$$

$$= \sum_{i=1}^{n} (|a_i| + |a_j|),$$

where a_j is the vertex of maximum Pythagorean fuzzy cardinality among all vertices other than a_i, $i = 1, 2, ..., n$. Hence,

$$\gamma_{Agt}(K_n) = \frac{\sum_{i=1}^{n} (|a_i| + |a_j|)}{|V|}.$$

Corollary 3.6. *Let $\mathcal{A}^{**}$ be the complete Pythagorean anti fuzzy graph with equal Pythagorean fuzzy vertex and edge cardinality. Then* $\gamma_{Ag}(\mathcal{A}^{**}) = \frac{2\sum_{i=1}^{n} |a_i|}{|V|}$.

Proof. By previous theorem, $\gamma_{Agt}(\mathcal{A}^{**}) = \frac{\sum_{i=1}^{n} (|a_i| + |a_j|)}{|V|}$.

Since $\mathcal{A}^{**}$ is the complete Pythagorean anti fuzzy graph with equal Pythagorean fuzzy vertex and edge cardinality,

$$\gamma_{Agt}(\mathcal{A}^{**}) = \frac{\sum_{i=1}^{n} (|a_i| + |a_i|)}{|V|} = \frac{2\sum_{i=1}^{n} |a_i|}{|V|}.$$

Theorem 3.7. *For any Pythagorean anti fuzzy graph $\mathcal{A}^{**}$,* $\gamma_{Agt}(\mathcal{A}^{**}) \leq \frac{2\sum_{i=1}^{n} |a_i|}{3}$.

Proof. Let $\mathcal{A}^{**}$ be the Pythagorean anti fuzzy graph and the vertex set be $\{a_1, a_2, ..., a_n\}$. Clearly, $\gamma_t(\mathcal{A}^{**}) \leq \frac{2\sum_{i=1}^{n} |a_i|}{3}$.

Let $\gamma_{at}(\mathcal{A}^{**})$ be the Pythagorean anti fuzzy total dominating number. Then

$$\gamma_{a_1 t}(\mathcal{A}^{**}) \leq \frac{2\sum_{i=1}^{n} | a_i |}{3},$$

$$\gamma_{a_2 t}(\mathcal{A}^{**}) \leq \frac{2\sum_{i=1}^{n} | a_i |}{3}, \ldots, \gamma_{a_n t}(\mathcal{A}^{**}) \leq \frac{2\sum_{i=1}^{n} | a_i |}{3},$$

$$\sum_{a \in V} \gamma_{at}(\mathcal{A}^{**}) \leq n\left[\frac{2\sum_{i=1}^{n} | a_i |}{3}\right].$$

Hence

$$\gamma_{Agt}(\mathcal{A}^{**}) \leq \frac{n\left[\frac{2\sum_{i=1}^{n} | a_i |}{3}\right]}{| V |} = \frac{2\sum_{i=1}^{n} | a_i |}{3}.$$

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