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ON FUZZY BAIRE SETS AND FUZZY PSEUDO-OPEN SETS

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Abstract

In this paper, several characterizations of fuzzy Baire sets and fuzzy pseudo-open sets in fuzzy topological spaces are established. It is obtained that fuzzy Baire sets in fuzzy hyperconnected and fuzzy second category (but not fuzzy Baire) spaces are fuzzy nowhere dense sets. It is observed that in fuzzy perfectly disconnected spaces, fuzzy regular closed sets and fuzzy residual sets are generating fuzzy Baire sets. Moreover, it is established that fuzzy pseudo-open sets are fuzzy simply* open sets in fuzzy *D*-Baire topological spaces.

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1. Introduction

The potential of the fuzzy notion introduced by Zadeh [13] in 1965 as a new approach to a mathematical representation of vagueness was realized by many researchers and it has successfully been applied in all branches of mathematics. In 1968, Chang [5] introduced the concept of a fuzzy topological space. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

In claasical topology, Baire [4] introduced the concepts of first category and second category sets and Denjoy [6] introduced residual sets which are the complements of first category sets. By means of first category sets, the notion of pseudo-open sets was introduced and studied by Neubrunnova [9]. In classical topology, Szymanski [1] defined sets of the form $U - E,$ where U is an open set and E is a meagre set in a topological space X as Baire sets of first type and $U \bigcup F,$ where U is an open set and F is a meagre set in X as Baire sets of second type.

The notion of fuzzy Baire sets was introduced by Thangaraj and Palani [25] in terms of fuzzy open sets and fuzzy residual sets in fuzzy topological spaces. The notion of fuzzy pseudo-open sets was introduced by Thangaraj and Dinakaran [21] in terms of fuzzy open sets and fuzzy first category sets in fuzzy topological spaces. The purpose of this paper is to study more deeply the notion of fuzzy Baire sets and fuzzy pseudo-open sets in fuzzy topological spaces.

In Section 3, several characterizations of fuzzy Baire sets are established. It is obtained that fuzzy Baire sets in fuzzy submaximal spaces and fuzzy globally disconnected spaces are fuzzy G_δ-sets, and fuzzy Baire sets in fuzzy hyperconnected and fuzzy second category (but not fuzzy Baire) spaces are fuzzy nowhere dense sets. It is shown that fuzzy regular closed sets and fuzzy residual sets in fuzzy perfectly disconnected spaces are generating fuzzy Baire sets and fuzzy open sets and complements of fuzzy σ-nowhere dense sets are generating fuzzy Baire sets in fuzzy topological spaces. It is obtained that fuzzy Baire sets in fuzzy strongly hyperconnected spaces are fuzzy open sets and fuzzy Baire sets in fuzzy extremally disconnected spaces are having fuzzy Baire sets as their super sets.

In Section 4, several characterizations of fuzzy pseudo-open sets are established. It is obtained that fuzzy pseudo-open sets in fuzzy *D*-Baire spaces are fuzzy simply* open sets and fuzzy pseudo-open sets in fuzzy hyperconnected and fuzzy *D*-Baire spaces are fuzzy simply open sets. In Section 5, conditions under which complements of fuzzy Baire sets become fuzzy pseudo-open sets and complements of fuzzy pseudo-open sets become fuzzy Baire sets, are obtained.

2. Preliminaries

We assume the basic definitions on fuzzy topology known to the readers [2, 4 -31]. However, to be as self contained as possible, we state some results that are used eventually in this paper. Throughout (X, T) represents a fuzzy topological space and λ a fuzzy set of X.

Theorem 2.1 [2]. *In a fuzzy topological space* (a) *the closure of a fuzzy open set is a fuzzy regular closed set*, *and* (b) *the interior of a fuzzy closed set is a fuzzy regular open set*.

Theorem 2.2 [10]. *If* λ *is a fuzzy residual set in* (X, T), *then there exists a fuzzy* G_δ*-set* μ *in* (X, T) *such that* $\mu \leq \lambda$.

Theorem 2.3 [22]. *If* λ *is a fuzzy regular closed set in a fuzzy perfectly disconnected space* (X, T), *then* λ *is a fuzzy open set in* (X, T).

Theorem 2.4 [17]. *If* λ *is a fuzzy* σ*-nowhere dense set in* (X, T), *then* $1 - \lambda$ *is a fuzzy residual set in* (X, T).

Theorem 2.5 [26]. *If* λ *is a fuzzy somewhere dense set in* (X, T), *then there exists a fuzzy regular closed set* η *in* (X, T) *such that* $\eta \leq cl(\lambda)$.

Theorem 2.6 [25]**.** *If* λ *is a fuzzy Baire set in* (X, T), *then there exists a fuzzy Baire set* β *in* (X, T) *such that* $\beta \leq \lambda$.

Theorem 2.7 [18]**.** *If* λ *is a fuzzy residual set in a fuzzy submaximal space* (X, T), *then* λ *is a fuzzy* G_δ*-set in* (X, T).

Theorem 2.8 [23]**.** *If* λ *is a fuzzy residual set in a fuzzy globally disconnected space* (X, T), *then* λ *is a fuzzy* G_δ*-set in* (X, T).

Theorem 2.9 [15]**.** *The following are equivalent*:

(1) (X, T) *is a fuzzy Baire space*.

(2) $Int(\lambda) = 0$, *for every fuzzy first category set* λ *in* (X, T).

(3) $Cl(\mu) = 1$, *for every fuzzy residual set* μ *in* (X, T).

Theorem 2.10 [10]**.** *If* $\lambda \leq \mu$ *and* λ *is a fuzzy residual set in* (X, T), *then* μ *is also a fuzzy residual set in* (X, T).

Theorem 2.11 [10]**.** *If* λ *is a fuzzy residual set in a fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then there exists a fuzzy closed fuzzy residual set* η *in* (X, T) *such that* $Cl(\lambda) \leq \eta$.

Theorem 2.12 [10]**.** *If a fuzzy topological space* (X, T) *is a fuzzy hyperconnected fuzzy second category* (*but not fuzzy Baire*) *space and* λ *is a fuzzy residual set in* (X, T), *then*

(i) λ *is a fuzzy nowhere dense set in* (X, T).

(ii) $1 - \lambda$ *is a fuzzy dense set in* (X, T).

Theorem 2.13 [10]**.** *If* λ *is a fuzzy residual set in a fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then* $\mathrm{int}(1 - \lambda)$ *is a non-zero fuzzy first category set in* (X, T).

Theorem 2.14 [16]. *If* (X, T) *is a fuzzy Baire fuzzy open hereditarily irresolvable space*, *then* (X, T) *is a fuzzy D-Baire space*.

Theorem 2.15 [20]. *If* λ *is a fuzzy simply* open set in a fuzzy hyperconnected space* (X, T), *then* λ *is a fuzzy simply open set in* (X, T).

Theorem 2.16 [10]. *If* λ *is a fuzzy first category set in* (X, T), *then there is a fuzzy first category set* μ *in* (X, T) *such that* $\lambda \leq \mu \leq cl(\lambda)$.

Theorem 2.17 [29]. *If* (X, T) *is a fuzzy strongly hyperconnected space*, *then* (X, T) *is a fuzzy hyperconnected fuzzy submaximal space*.

Theorem 2.18 [24]. *If* λ *is a fuzzy open fuzzy dense set in* (X, T), *then* $1 - \lambda$ *is a fuzzy resolvable set in* (X, T).

Theorem 2.19 [27]. *If* λ *is a fuzzy open set in a fuzzy Baire space* (X, T), *then* λ *is a fuzzy Baire dense set in* (X, T).

Theorem 2.20 [30]. *If* μ *is a fuzzy residual set in a fuzzy almost P-space* (X, T), *then* μ *is a fuzzy somewhere dense set in* (X, T).

3. Fuzzy Baire Sets in Fuzzy Topological Spaces

Motivated by the works of Neubrunnova [9] and Szymanski [1] in classical topology, the concept of fuzzy Baire sets is introduced and studied in fuzzy topology [25]. The purpose of this section is to study more deeply the notion of fuzzy Baire sets in fuzzy topological spaces.

Definition 3.1 [25]. A fuzzy set λ in (X, T) is called a *fuzzy Baire set* if $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T).

Example 3.1. Let $X = \{A, B, C\}$. Consider the following fuzzy sets $\lambda, \mu, \alpha, \beta, \theta$ and δ defined on *X*.

$\lambda : X \to I$ is defined by $\lambda(A) = 1$; $\lambda(B) = 0.2$; $\lambda(C) = 0.7$;

$\mu : X \to I$ is defined by $\mu(A) = 0.7$; $\mu(B) = 0.4$; $\mu(C) = 1$;

$\alpha : X \to I$ is defined by $\alpha(A) = 0.7$; $\alpha(B) = 0.3$; $\alpha(C) = 0.7$;

$\beta : X \to I$ is defined by $\beta(A) = 0.8$; $\beta(B) = 0.6$; $\beta(C) = 0.9$;

$\theta : X \to I$ is defined by $\theta(A) = 0.8$; $\theta(B) = 0.9$; $\theta(C) = 0.7$;

$\delta : X \to I$ is defined by $\delta(A) = 0.7$; $\delta(B) = 0.4$; $\delta(C) = 0.7$.

Then, $T = \{0, \lambda, \mu, \lambda \vee \mu, \lambda \vee \mu, 1\}$ is a fuzzy topology on X. By computation,

$cl(\lambda) = 1$; $cl(\mu) = 1$; $cl(\lambda \vee \mu) = 1$; $cl(\lambda \wedge \mu) = 1$; $cl(\alpha) = 1$;

$cl(\beta) = 1$; $cl(\theta) = 1$; $cl(\delta) = 1$; $\operatorname{int}(1 - \lambda) = 0$; $\operatorname{int}(1 - \mu) = 0$;

$\operatorname{int}(1 - [\lambda \vee \mu]) = 0$; $\operatorname{int}(1 - [\lambda \wedge \mu]) = 0$; $\operatorname{int}(\alpha) = \lambda \wedge \mu$;

$\operatorname{int}(\beta) = \lambda \wedge \mu$; $\operatorname{int}(\theta) = \lambda \wedge \mu$; $\operatorname{int}(\delta) = \lambda \wedge \mu$; $\operatorname{int}(1 - \alpha) = 0$;

$\operatorname{int}(1 - \beta) = 0$; $\operatorname{int}(1 - \theta) = 0$; $\operatorname{int}(1 - \delta) = 0$.

Now

$cl\operatorname{int}(\alpha) = 1$; $cl\operatorname{int}(\beta) = 1$; $cl\operatorname{int}(\theta) = 1$; $cl\operatorname{int}(\delta) = 1$; and

$cl\operatorname{int}(\lambda) = 1$; $cl\operatorname{int}(\mu) = 1$; $cl\operatorname{int}(\lambda \vee \mu) = 1$; $cl\operatorname{int}(\lambda \wedge \mu) = 1$.

Further,

$$\alpha = \alpha \wedge \beta \wedge \theta \wedge \delta \quad \text{and} \quad \lambda \wedge \mu = \lambda \wedge \mu \wedge (\lambda \vee \mu) \wedge (\lambda \wedge \mu).$$

Thus, α and $\lambda \wedge \mu$ are fuzzy residual sets in (X, T). (It should be noted that the fuzzy residual set α is not a fuzzy open set whereas the fuzzy residual set $\lambda \wedge \mu$ is a fuzzy open set in (X, T).) On computing, $\lambda \wedge \alpha = \lambda \wedge \mu$; $\mu \wedge \alpha = \alpha$; $(\lambda \vee \mu) \wedge \alpha = \alpha$; $(\lambda \wedge \mu) \wedge \alpha = \lambda \wedge \mu$ and $\lambda \wedge (\lambda \wedge \mu)$

$= \lambda \wedge \mu$; $\mu \wedge (\lambda \wedge \mu) = \lambda \wedge \mu$; $(\lambda \vee \mu) \wedge (\lambda \wedge \mu) = \lambda \wedge \mu$; $(\lambda \wedge \mu) \wedge (\lambda \wedge \mu) = \lambda \wedge \mu$. Hence α and $\lambda \wedge \mu$ are the fuzzy Baire sets in (X, T). (It should be noted that the fuzzy Baire set α is not a fuzzy open set in (X, T).)

Proposition 3.1. *If a fuzzy set* λ *is a fuzzy Baire set in* (X, T), *then there exists a fuzzy* G_δ*-set* θ *in* (X, T) *such that* $\theta \leq \lambda$.

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). By Theorem 2.2, for the fuzzy residual set η, there exists a fuzzy G_δ-set α in (X, T) such that $\alpha \leq \eta$. Then, $\mu \wedge \alpha \leq \mu \wedge \eta$ in (X, T). Now $\alpha = \wedge_{i=1}^{\infty}(\alpha_i)$, where $\alpha_i \in T$ and μ is a fuzzy open set in (X, T), $\mu \wedge \alpha = \mu \wedge \wedge_{i=1}^{\infty}(\alpha_i)$ is a fuzzy G_δ-set in (X, T). Let $\theta = \mu \wedge \alpha$. Then θ is a fuzzy G_δ-set in (X, T). Hence, for the fuzzy Baire set λ in (X, T), there exists a fuzzy G_δ-set θ in (X, T) such that $\theta \leq \lambda$.

Corollary 3.1. *If a fuzzy set* λ *is a fuzzy Baire set in* (X, T), *then there exists a fuzzy* F_σ*-set* δ *in* (X, T) *such that* $1 - \lambda \leq \delta$.

Proof. Let λ be a fuzzy Baire set in (X, T). By Proposition 3.1, there exists a fuzzy G_δ-set θ in (X, T) such that $\theta \leq \lambda$. Then, $1 - \lambda \leq 1 - \theta$, in (X, T). Let $\delta = 1 - \theta$. Then, δ is a fuzzy F_σ-set δ in (X, T) such that $1 - \lambda \leq \delta$.

Proposition 3.2. *If a fuzzy set* λ *is a fuzzy Baire set in a fuzzy Baire space* (X, T), *then* $\lambda = \mu \wedge \eta$, *where* μ *is a Baire dense set and* η *is a fuzzy residual set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Because (X, T) is a fuzzy Baire space, by Theorem 2.19, the fuzzy open set μ is a

Baire dense set in (X, T). Hence $\lambda = \mu \wedge \eta$, where μ is a Baire dense set and η is a fuzzy residual set in (X, T).

Proposition 3.3. *If a fuzzy set* λ *is a fuzzy Baire set in a fuzzy strongly hyperconnected space* (X, T), *then* λ *is a fuzzy open set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Now $1 - \eta$ is a fuzzy first category set in (X, T). Since (X, T) is a fuzzy strongly hyperconnected space, by Theorem 2.17, (X, T) is a fuzzy hyperconnected fuzzy submaximal space. By Theorem 2.16, there is a fuzzy first category set μ in (X, T) such that $1 - \eta \leq \mu \leq cl(1 - \eta)$. Now $\mu \leq cl(1 - \eta)$, which implies that $\mu \leq 1 - \operatorname{int}(\eta)$. Thus, $\operatorname{int}(\eta) \leq 1 - \mu$. This implies that $cl[\operatorname{int}(\eta)] \leq cl(1 - \mu)$. Since $\operatorname{int}(\eta)$ is a fuzzy open set in the fuzzy hyperconnected space (X, T), $cl[\operatorname{int}(\eta)] = 1$ and thus $cl(1 - \mu) = 1$. Now $1 - \eta \leq \mu$, which implies that $1 - \mu \leq \eta$ and thus $cl(1 - \mu) \leq cl(\eta)$ and $1 \leq cl(\eta)$. That is, $cl(\eta) = 1$, in (X, T). Since (X, T) is a fuzzy submaximal space, $cl(\eta) = 1$, which implies that η is a fuzzy open set in (X, T). Thus, $\lambda = \mu \wedge \eta$, where $\mu \in T$ and $\eta \in T$, implying that λ is a fuzzy open set in (X, T).

Example 3.2. Let $X = \{A, B, C\}$. Consider the following fuzzy sets α, β, γ and δ defined on X:

$\alpha : X \to I$ is defined by $\alpha(A) = 0.6$; $\alpha(B) = 0.9$; $\alpha(C) = 0.8$,

$\beta : X \to I$ is defined by $\beta(A) = 0.7$; $\beta(B) = 0.8$; $\beta(C) = 0.9$,

$\gamma : X \to I$ is defined by $\gamma(A) = 0.8$; $\gamma(B) = 0.6$; $\gamma(C) = 0.7$.

Then,

$$T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \alpha \vee [\beta \wedge \gamma],$$

$$\gamma \vee [\alpha \wedge \beta], \beta \wedge [\alpha \vee \gamma], \alpha \vee \beta \vee \gamma, 1\}$$

is a fuzzy topology on X. By computation, $cl(\alpha) = 1$; $cl(\beta) = 1$; $cl(\gamma) = 1$; $cl(\alpha \vee \beta) = 1$; $cl(\alpha \vee \gamma) = 1$; $cl(\beta \vee \gamma) = 1$; $cl(\alpha \wedge \beta) = 1$; $cl(\alpha \wedge \gamma) = 1$; $cl(\beta \wedge \gamma) = 1$; $cl(\alpha \vee [\beta \wedge \gamma]) = 1$; $cl(\gamma \vee [\alpha \wedge \beta]) = 1$; $cl(\beta \wedge [\alpha \vee \gamma]) = 1$ and $cl(\alpha \vee \beta \vee \gamma) = 1$, in (X, T). It is observed that all the fuzzy open sets in (X, T) are fuzzy dense sets and thus (X, T) is a fuzzy hyperconnected space. Also, fuzzy dense sets are fuzzy open sets in (X, T) which implies that (X, T) is a fuzzy submaximal space and hence (X, T) is a fuzzy strongly hyperconnected space.

Since all the fuzzy open sets are fuzzy dense, their complements $1 - \alpha$, $1 - \beta$, $1 - \gamma$, $1 - (\alpha \vee \beta)$, $1 - (\alpha \vee \gamma)$, $1 - (\beta \vee \gamma)$, $1 - (\alpha \wedge \beta)$, $1 - (\alpha \wedge \gamma)$, $1 - (\beta \wedge \gamma)$, $1 - (\alpha \vee [\beta \wedge \gamma])$, $1 - (\gamma \vee [\alpha \wedge \beta])$, $1 - (\beta \wedge [\alpha \vee \gamma])$ and $1 - (\alpha \vee \beta \vee \gamma)$ are fuzzy nowhere dense sets in (X, T). Further, fuzzy first category sets in (X, T) are $1 - \gamma$, $1 - (\alpha \wedge \gamma)$ and $1 - (\beta \wedge [\alpha \vee \gamma])$ and fuzzy residual sets in (X, T) are γ, $(\alpha \wedge \gamma)$ and $(\beta \wedge [\alpha \vee \gamma])$. As fuzzy Baire dense sets in (X, T) are γ, $\alpha \wedge \beta$, $\alpha \wedge \gamma$, $\beta \wedge \gamma$, these are fuzzy open in (X, T).

Proposition 3.4. *If* λ *is a fuzzy Baire set in a fuzzy D-Baire space* (X, T), *then there exists a fuzzy regular closed set* δ *in* (X, T) *such that* $\lambda \leq \delta$.

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Now $1 - \eta$ is a fuzzy first category set in (X, T). Because (X, T) is a fuzzy *D*-Baire space, $1-\eta$ is a fuzzy nowhere dense set and thus $\operatorname{int} cl(1-\eta)=0$, in (X, T). Because $\operatorname{int}(1-\eta) \leq \operatorname{int} cl(1-\eta)$, $\operatorname{int}(1-\eta)=0$, by Lemma 2.1, $1-cl(\eta)=\operatorname{int}(1-\eta)=0$ and thus $cl(\eta)=1$, in (X, T). Now $\lambda=\mu \wedge \eta$, which implies that $cl(\lambda)=cl(\mu \wedge \eta) \leq cl(\mu) \wedge cl(\eta)=cl(\mu) \wedge 1=cl(\mu)$. Let $\delta=cl(\mu)$. By Theorem 2.1, $cl(\mu)$ is a fuzzy regular closed set. Let $\delta=cl(\mu)$ and $\lambda \leq cl(\lambda) \leq \delta$. Then, for the fuzzy Baire set λ in (X, T), there exists a fuzzy regular closed set δ in (X, T) such that $\lambda \leq \delta$.

Example 3.3. Let $X=\{A, B, C\}$. Consider the following fuzzy sets α, β, γ and θ defined on X.

$\alpha: X \rightarrow I$ is defined by $\alpha(A)=0.5$; $\alpha(B)=0.3$; $\alpha(C)=0.5$,

$\beta: X \rightarrow I$ is defined by $\beta(A)=0.6$; $\beta(B)=0.5$; $\beta(C)=0.7$,

$\gamma: X \rightarrow I$ is defined by $\gamma(A)=0.5$; $\gamma(B)=0.4$; $\gamma(C)=0.6$,

$\theta: X \rightarrow I$ is defined by $\theta(A)=0.5$; $\theta(B)=0.6$; $\theta(C)=0.4$.

Then, $T=\{0, \alpha, \beta, \gamma, 1\}$ is a fuzzy topology on X. By computation, $cl(\alpha)=1-\alpha$; $cl(\beta)=1$; $cl(\gamma)=1$ and $\operatorname{int}(1-\alpha)=\alpha$; $\operatorname{int}(1-\beta)=0$; $\operatorname{int}(1-\gamma)=0$, in (X, T). Now $\operatorname{int} cl(1-\alpha)=\alpha$; $\operatorname{int} cl(1-\beta)=0$ and $\operatorname{int} cl(1-\gamma)=0$. Thus fuzzy nowhere dense sets in (X, T) are $1-\beta$ and $1-\gamma$. Further, $\theta=(1-\beta) \vee(1-\gamma)$ and thus θ is a fuzzy first category set in (X, T). Also, $\operatorname{int} cl(\theta)=\operatorname{int}(1-\gamma)=0$. Thus, the fuzzy first category set γ is a fuzzy nowhere dense set in (X, T), which implies that (X, T) is a fuzzy D-Baire space.

Now $1-\theta=\gamma$ is a fuzzy residual set in (X, T) and $\alpha \wedge \gamma=\alpha$ and $\beta \wedge \gamma=\gamma$, implying that α and γ are fuzzy Baire sets in (X, T). By computation, $cl \operatorname{int}(1-\alpha)=cl(\alpha)=1-\alpha$ and thus $1-\alpha$ is a fuzzy regular closed set in (X, T), $\alpha \leq (1 - \alpha)$, in (X, T) and $cl\,\mathrm{int}(1) = 1$ and $\gamma \leq 1$, in (X, T).

Proposition 3.5. *If* $\lambda = \mu \wedge \eta$, *where* μ *is a fuzzy regular closed set and* η *is a fuzzy residual set in a fuzzy perfectly disconnected space* (X, T), *then* λ *is a fuzzy Baire set in* (X, T).

Proof. Suppose that for a λ defined on X, $\lambda = \mu \wedge \eta$, where μ is a fuzzy regular closed set and η is a fuzzy residual set in (X, T). Since (X, T) is a fuzzy perfectly disconnected space, by Theorem 2.3, the fuzzy regular closed set μ is a fuzzy open set in (X, T). Hence $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T), implying that λ is a fuzzy Baire set in (X, T).

Proposition 3.6. *If* $\lambda = \mu \wedge [1 - \eta]$, *where* μ *is a fuzzy open set and* η *is a fuzzy* σ*-nowhere dense set in a fuzzy topological space* (X, T), *then the fuzzy set* λ *is a fuzzy Baire set in* (X, T).

Proof. Suppose for a λ defined on X, $\lambda = \mu \wedge [1 - \eta]$, where μ is a fuzzy open set and η is a fuzzy σ-nowhere dense set in (X, T). By Theorem 2.4, for the fuzzy σ-nowhere dense set η, $1 - \eta$ is a fuzzy residual set in (X, T). Hence $\lambda = \mu \wedge [1 - \eta]$, where μ is a fuzzy open set and $1 - \eta$ is a fuzzy residual set in (X, T), implying that λ is a fuzzy Baire set in (X, T).

Corollary 3.2. *If* $\lambda = \mu \wedge \gamma$, *where* μ *is a fuzzy open set and* γ *is a fuzzy* G_δ*-set with* $cl(\gamma) = 1$, *in* (X, T), *then the fuzzy set* λ *is a fuzzy Baire set in* (X, T).

Proof. Suppose for a λ defined on X, $\lambda = \mu \wedge \gamma$, where μ is a fuzzy open set and γ is a fuzzy G_δ-set set with $cl(\gamma) = 1$, in (X, T). By Lemma

2.1, $\operatorname{int}(1-\gamma) = 1 - cl(\gamma)$ and then $\operatorname{int}(1-\gamma) = 1 - 1 = 0$. Let $\gamma = 1 - \eta$. Then, $\eta = 1 - \gamma$ and η is a fuzzy F_σ-set with $\operatorname{int}(\eta) = 0$. This implies that η is a fuzzy σ-nowhere dense set in (X, T). Hence $\lambda = \mu \wedge [1 - \eta]$, where μ is a fuzzy open set and η is a fuzzy σ-nowhere dense set in (X, T). By Proposition 3.6, λ is a fuzzy Baire set in (X, T).

Proposition 3.7. *If λ is a fuzzy somewhere dense set in a fuzzy perfectly disconnected space (X, T), then there exists a fuzzy Baire set δ in (X, T) such that $\delta \le cl(\lambda) \wedge \eta$, where η is a fuzzy residual set in (X, T).*

Proof. Let λ be a fuzzy somewhere dense set in (X, T). Then, by Theorem 2.5, there exists a fuzzy regular closed set μ in (X, T) such that $\mu \le cl(\lambda)$. Thus, $\mu \wedge \eta \le cl(\lambda) \wedge \eta$, where η is a fuzzy residual set in (X, T). Let $\delta = \mu \wedge \eta$. Since (X, T) is a fuzzy perfectly disconnected space, by Theorem 2.3, the fuzzy regular closed set μ is a fuzzy open set in (X, T). Hence $\delta = \mu \wedge \eta$, where $\mu \in T$ and η is a fuzzy residual set in (X, T), implying that δ is a fuzzy Baire set in (X, T). Hence, for the fuzzy somewhere dense set λ, there exists a fuzzy Baire set δ in (X, T) such that $\delta \le cl(\lambda) \wedge \eta$, where η is a fuzzy residual set in (X, T).

Proposition 3.8. *If λ is a fuzzy Baire set in a fuzzy extremally disconnected space (X, T), then there exists a fuzzy Baire set δ in (X, T) such that $\lambda \le \delta$.*

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Now $\mu \le cl(\mu)$, which implies that $\mu \wedge \eta \le cl(\mu) \wedge \eta$, in (X, T). Since (X, T) is a fuzzy extremally disconnected space, for the fuzzy open set μ, $cl(\mu)$ is a fuzzy open set in (X, T). Let $\delta = cl(\mu) \wedge \eta$. Then, δ is a fuzzy Baire set in (X, T) such that $\lambda \le \delta$.

Remark 3.1. In Example 3.1, for fuzzy open sets λ, μ, $\lambda \vee \mu$, $\lambda \wedge \mu$, $cl(\lambda) = 1 \in T$; $cl(\mu) = 1 \in T$; $cl(\lambda \vee \mu) = 1 \in T$; $cl(\lambda \wedge \mu) = 1 \in T$. Thus, (X, T) is a fuzzy extremally disconnected space. By computation, α and $\lambda \wedge \mu$ are fuzzy Baire sets in (X, T) and $\lambda \wedge \mu \leq \alpha$, in (X, T).

Corollary 3.3. *If λ is a fuzzy Baire set in a fuzzy extremally disconnected space (X, T), then there exist fuzzy Baire sets δ_1 and δ_2 in (X, T) such that $\delta_1 \leq \lambda \leq \delta_2$.*

Proof. The proof follows from Proposition 3.8 and Theorem 2.6.

The following proposition ensures that fuzzy Baire sets are fuzzy G_δ-sets in fuzzy submaximal spaces.

Proposition 3.9. *If a fuzzy set λ is a fuzzy Baire set in a fuzzy submaximal space (X, T), then λ is a fuzzy G_δ-set in (X, T).*

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Since (X, T) is a fuzzy submaximal space, by Theorem 2.7, the fuzzy residual set η is a fuzzy G_δ-set in (X, T). Then, $\eta = \wedge_{i=1}^{\infty}(\alpha_i)$, where $\alpha_i \in T$. Now $\lambda = \mu \wedge \eta = \mu \wedge \wedge_{i=1}^{\infty}(\alpha_i)$ and thus λ is a fuzzy G_δ-set in (X, T).

Example 3.4. Let $X = \{A, B, C\}$. Consider the following fuzzy sets α, β and γ defined on X:

$\alpha : X \to I$ is defined by $\alpha(A) = 0.6$; $\alpha(B) = 0.9$; $\alpha(C) = 0.8$,

$\beta : X \to I$ is defined by $\beta(A) = 0.7$; $\beta(B) = 0.8$; $\beta(C) = 0.9$,

$\gamma : X \to I$ is defined by $\gamma(A) = 0.8$; $\gamma(B) = 0.6$; $\gamma(C) = 0.7$.

Then,

$$T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \alpha \vee [\beta \wedge \gamma],$$
$$\gamma \vee [\alpha \wedge \beta], \beta \wedge [\alpha \vee \gamma], \alpha \vee \beta \vee \gamma, 1\}$$

is a fuzzy topology on X. By computation, $cl(\alpha) = 1$; $cl(\beta) = 1$; $cl(\gamma) = 1$; $cl(\alpha \vee \beta) = 1$; $cl(\alpha \vee \gamma) = 1$; $cl(\beta \vee \gamma) = 1$; $cl(\alpha \wedge \beta) = 1$; $cl(\alpha \wedge \gamma) = 1$; $cl(\beta \wedge \gamma) = 1$; $cl(\alpha \vee [\beta \wedge \gamma]) = 1$; $cl(\gamma \vee [\alpha \wedge \beta]) = 1$; $cl(\beta \wedge [\alpha \vee \gamma]) = 1$ and $cl(\alpha \vee \beta \vee \gamma) = 1$, in (X, T). It is observed that all fuzzy dense sets are fuzzy open sets in (X, T) which implies that (X, T) is a fuzzy submaximal space. Further, fuzzy G_δ-sets in (X, T) are γ, $\alpha \wedge \beta$, $\alpha \wedge \gamma$, $\beta \wedge \gamma$ and $\beta \wedge [\alpha \vee \gamma]$. Also, fuzzy Baire sets in (X, T) are γ, $\alpha \wedge \beta$, $\alpha \wedge \gamma$, $\beta \wedge \gamma$ and $\beta \wedge [\alpha \vee \gamma]$. It is observed that all fuzzy Baire sets are fuzzy G_δ-sets in (X, T).

Corollary 3.4. *If* λ *is a fuzzy Baire set in a fuzzy submaximal space* (X, T), *then* $1 - \lambda$ *is a fuzzy* F_σ*-set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Then, by Proposition 3.9, λ is a fuzzy G_δ-set in (X, T) and thus the fuzzy set $1 - \lambda$ is a fuzzy F_σ-set in (X, T).

Proposition 3.10. *If* λ *is a fuzzy Baire set in a fuzzy globally disconnected space* (X, T), *then* λ *is a fuzzy* G_δ*-set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Since (X, T) is a fuzzy globally disconnected space, by Theorem 2.8, the fuzzy residual set η is a fuzzy G_δ-set in (X, T). Then, $\eta = \wedge_{i=1}^{\infty}(\alpha_i)$, where $\alpha_i \in T$. Now $\lambda = \mu \wedge \eta = \mu \wedge \wedge_{i=1}^{\infty}(\alpha_i)$ and thus λ is a fuzzy G_δ-set in (X, T).

Corollary 3.5. *If* λ *is a fuzzy Baire set in a fuzzy globally disconnected space* (X, T), *then* $1 - \lambda$ *is a fuzzy* F_σ*-set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Then, by Proposition 3.10, λ is a fuzzy G_δ-set in (X, T) and thus the fuzzy set $1 - \lambda$ is a fuzzy F_σ-set in (X, T).

Proposition 3.11. *If* λ *is a fuzzy Baire set in a fuzzy globally disconnected fuzzy almost P-space* (X, T), *then* $\lambda = \mu \wedge \eta$, *where* μ *is a fuzzy open set and* η *is a fuzzy somewhere dense set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Since (X, T) is a fuzzy globally disconnected space, by Theorem 2.8, the fuzzy residual set η is a fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy almost *P*-space, $\operatorname{int}(\eta) \neq 0$, in (X, T). Now $\operatorname{int}(\eta) \leq \operatorname{int} cl(\eta)$, which implies that $\operatorname{int} cl(\eta) \neq 0$, in (X, T). Hence η is a fuzzy somewhere dense set in (X, T). Thus, it follows that $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy somewhere dense set in (X, T).

Proposition 3.12. *If* λ *is a fuzzy Baire set in a fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then there exists a fuzzy Baire set* θ *in* (X, T) *such that* $\lambda \leq \mu \wedge cl(\eta) \leq \theta$, *where* μ *is a fuzzy open set and* η *is a fuzzy residual set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Because (X, T) is a fuzzy second category (but not fuzzy Baire) space, by Theorem 2.11, for the fuzzy residual set η in (X, T), there exists a fuzzy closed fuzzy residual set β in (X, T) such that $cl(\eta) \leq \beta$. Then, $\eta \leq cl(\eta) \leq \beta$ implies that $\mu \wedge \eta \leq \mu \wedge cl(\eta) \leq \mu \wedge \beta$ and $\lambda \leq \mu \wedge cl(\eta) \leq \mu \wedge \beta$. Let $\theta = \mu \wedge \beta$.

Since μ is a fuzzy open set and β is a fuzzy residual set in (X, T), θ is a fuzzy Baire set in (X, T). Thus it follows that $\lambda \leq \mu \wedge cl(\eta) \leq \theta$.

Example 3.5. Let $X = \{A, B, C\}$. Consider the following fuzzy sets α, β and γ defined on X:

$\alpha : X \to I$ is defined by $\alpha(A) = 1;\quad \alpha(B) = 0.2;\quad \alpha(C) = 0.7,$

$\beta : X \to I$ is defined by $\beta(A) = 0.3;\quad \beta(B) = 1;\quad \beta(C) = 0.2,$

$\gamma : X \to I$ is defined by $\gamma(A) = 0.7;\quad \gamma(B) = 0.4;\quad \gamma(C) = 1.$

Then,

$$T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \alpha \vee [\beta \wedge \gamma], \beta \vee [\alpha \wedge \gamma], \gamma \wedge [\alpha \vee \beta], \alpha \vee \beta \vee \gamma, 1\}$$

is a fuzzy topology on X.

By computation,

$$cl(\alpha) = 1;\quad cl(\beta) = 1;\quad cl(\gamma) = 1;\quad cl(\alpha \vee \beta) = 1;\quad cl(\alpha \vee \gamma) = 1;$$

$$cl(\beta \vee \gamma) = 1;\quad cl(\alpha \wedge \beta) = 1 - (\gamma \wedge [\alpha \vee \beta]);\quad cl(\alpha \wedge \gamma) = 1 - (\beta \wedge \gamma);$$

$$cl(\beta \wedge \gamma) = 1 - (\gamma \wedge [\alpha \vee \beta]);\quad cl(\beta \wedge \gamma) = 1 - (\gamma \wedge [\alpha \vee \beta]);$$

$$cl(\alpha \vee [\beta \wedge \gamma]) = 1;\quad cl(\beta \vee [\alpha \wedge \gamma]) = 1;\quad cl(\gamma \wedge [\alpha \vee \beta]) = 1 - (\beta \wedge \gamma)$$

and $cl(\alpha \vee \beta \vee \gamma) = 1$, in (X, T). Furthher, the fuzzy nowhere dense sets in (X, T) are $1 - \alpha$, $1 - \beta$, $1 - \gamma$, $1 - (\alpha \vee \beta)$, $1 - (\alpha \vee \gamma)$, $1 - (\beta \vee \gamma)$, $1 - (\alpha \vee [\beta \wedge \gamma])$ and $1 - (\beta \vee [\alpha \wedge \gamma])$. Also,

$$1 - (\alpha \wedge \beta) = (1 - \alpha) \vee (1 - \beta) \vee (1 - \gamma) \vee (1 - (\alpha \vee \beta)) \vee (1 - (\alpha \vee \gamma)) \vee (1 - (\beta \vee \gamma)),$$

$$1 - (\gamma \wedge [\alpha \vee \beta]) = (1 - \gamma) \vee (1 - (\alpha \vee \beta)) \vee (1 - (\alpha \vee \gamma)) \vee (1 - (\beta \vee \gamma)).$$

Thus, fuzzy first category sets in (X, T) are $(1 - \alpha \wedge \beta)$, $1 - (\gamma \wedge [\alpha \vee \beta])$ and hence $\alpha \wedge \beta$ and $\gamma \wedge [\alpha \vee \beta]$ are fuzzy residual sets in (X, T). By computation, $\alpha \wedge \beta$, $\alpha \wedge \gamma$, $\beta \wedge \gamma$ and $\gamma \wedge [\alpha \vee \beta]$ are fuzzy Baire sets in (X, T). Following can easily be verified:

(i) $\alpha \wedge \beta \leq \alpha \wedge cl(\alpha \wedge \beta) \leq \gamma \wedge [\alpha \vee \beta]$, where $\alpha \in T$ and $\alpha \wedge \beta$ is a fuzzy residual set in (X, T).

(ii) $\gamma \wedge [\alpha \vee \beta] \leq (\alpha \vee [\beta \wedge \gamma]) \wedge cl(\gamma \wedge [\alpha \vee \beta]) \leq \gamma \wedge [\alpha \vee \beta]$, where $\alpha \vee [\beta \wedge \gamma] \in T$ and $\gamma \wedge [\alpha \vee \beta]$ is a fuzzy residual set in (X, T).

(iii) $\beta \wedge \gamma \leq (\alpha \vee [\beta \wedge \gamma]) \wedge cl(\alpha \wedge \beta) \leq \gamma \wedge [\alpha \vee \beta]$, where $\alpha \vee [\beta \wedge \gamma] \in T$ and $\alpha \wedge \beta$ is a fuzzy residual set in (X, T).

(iv) $\alpha \wedge \gamma \leq (\alpha \vee [\beta \wedge \gamma]) \wedge cl(\gamma \wedge [\alpha \vee \beta]) \leq \gamma \wedge [\alpha \vee \beta]$, where $\alpha \vee [\beta \wedge \gamma] \in T$ and $\gamma \wedge [\alpha \vee \beta]$ is a fuzzy residual set in (X, T).

Remark 3.2. It is observed that fuzzy Baire sets in fuzzy hyperconnected spaces need not be fuzzy open. For, in Example 3.1, $cl(\lambda) = 1$; $cl(\mu) = 1$; $cl(\lambda \vee \mu) = 1$; $cl(\lambda \wedge \mu) = 1$, in (X, T). Since all the fuzzy open sets are fuzzy dense sets in (X, T), (X, T) is a fuzzy hyperconnected space.

By computation, $\lambda \wedge \mu$ and α are fuzzy residual sets in (X, T) in which $\lambda \wedge \mu \in T$ whereas $\alpha \notin T$. Fuzzy Baire sets in (X, T) are $\lambda \wedge \mu$ $[= (\lambda \vee \mu) \wedge (\lambda \wedge \mu)]$ and α $[= (\lambda \vee \mu) \wedge \alpha]$. It should be noted that the fuzzy Baire set α is not a fuzzy open set in (X, T) and also is not a fuzzy closed set in (X, T).

The following proposition shows that fuzzy Baire sets in fuzzy hyperconnected fuzzy second category (but no fuzzy Baire) spaces are fuzzy nowhere dense sets.

Proposition 3.13. *If* λ *is a fuzzy Baire set in a fuzzy hyperconnected fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then* λ *is a fuzzy nowhere dense set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Since (X, T) is a fuzzy hyperconnected fuzzy second category (but not fuzzy Baire) space, by Theorem 2.12, the fuzzy residual set η is a fuzzy nowhere dense set in (X, T). Then, $\operatorname{int} cl(\eta) = 0$, in (X, T). Now

$$\operatorname{int} cl(\lambda) = \operatorname{int} cl(\mu \wedge \eta) \leq \operatorname{int}[cl(\mu) \wedge cl(\eta)]$$

$$= \operatorname{int} cl(\mu) \wedge \operatorname{int} cl(\eta) = \operatorname{int} cl(\mu) \wedge 0 = 0$$

and then $\operatorname{int} cl(\lambda) = 0$. Hence λ is a fuzzy nowhere dense set in (X, T).

Corollary 3.6. *If* λ *is a fuzzy Baire set in a fuzzy hyperconnected fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then* $1 - cl(\lambda)$ *is a fuzzy open fuzzy dense set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Since (X, T) is a fuzzy hyperconnected fuzzy second category (but not fuzzy Baire) space, by Proposition 3.13, λ is a fuzzy nowhere dense set in (X, T). Then, $\operatorname{int} cl(\lambda) = 0$, in (X, T). Now $cl(\lambda)$ is a fuzzy closed set in (X, T), implying that $1 - cl(\lambda)$ is a fuzzy open set in (X, T). Also, $cl[1 - cl(\lambda)] = 1 - \operatorname{int}[cl(\lambda)] = 1 - 0 = 1$, which implies that $1 - cl(\lambda)$ is a fuzzy dense set in (X, T). Thus, $1 - cl(\lambda)$ is a fuzzy open fuzzy dense set in (X, T).

Proposition 3.14. *If* λ *is a fuzzy Baire set in a fuzzy hyperconnected fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then* $cl(\lambda)$ *is a fuzzy resolvable set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Since (X, T) is a fuzzy hyperconnected fuzzy second category (but not fuzzy Baire) space (X, T), by Corollary 3.6, $1 - cl(\lambda)$ is a fuzzy open fuzzy dense set in (X, T). By Theorem 2.18, $1 - [1 - cl(\lambda)]$ is a fuzzy resolvable set in (X, T). Hence, $cl(\lambda)$ is a fuzzy resolvable set in (X, T).

Proposition 3.15. *If* (λ_i)*'s* $(i = 1$ *to* $\infty)$ *are fuzzy Baire sets in a fuzzy hyperconnected fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then* $\operatorname{int}(\vee_{i=1}^{\infty}(\lambda_i)) \neq 0$, *in* (X, T).

Proof. Let (λ_i)'s $(i = 1$ to $\infty)$ be fuzzy Baire sets in (X, T). Because (X, T) is a fuzzy hyperconnected fuzzy second category (but not fuzzy Baire) space, by Proposition 3.13, (λ_i)'s are fuzzy nowhere dense sets in (X, T). Then, $\vee_{i=1}^{\infty}(\lambda_i)$ is a fuzzy first category set in (X, T). Because (X, T) is not a fuzzy Baire space, it follows, by Theorem 2.9, that $\operatorname{int}(\vee_{i=1}^{\infty}(\lambda_i)) \neq 0$, in (X, T).

Proposition 3.16. *If* λ *is a fuzzy Baire set in a fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then* $\lambda = \mu \wedge \eta$, *where* μ *is a fuzzy open set and* η *is a fuzzy cs dense set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Since (X, T) is fuzzy second category (but not fuzzy Baire) space, by Theorem 2.13, $\operatorname{int}(1 - \eta)$ is a fuzzy first category set in (X, T). Since (X, T) is not a fuzzy Baire space, it follows, by Theorem 2.9, that $\operatorname{int}(1 - \eta) \neq 0$. Now $\operatorname{int}(1 - \eta) \leq \operatorname{int} cl(1 - \eta)$, which implies that $\operatorname{int} cl(1 - \eta) \neq 0$ and thus $1 - \eta$ is a fuzzy somewhere dense set in (X, T). Then, η is a fuzzy cs dense set in (X, T). Hence $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy cs dense set in (X, T).

Proposition 3.17. *If there exists a fuzzy Baire set in a fuzzy hyperconnected fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then* (X, T) *is not a fuzzy hereditarily irresolvable space.*

Proof. Suppose λ is a fuzzy Baire set in (X, T). Because (X, T) is a fuzzy hyperconnected fuzzy second category (but not fuzzy Baire) space, by Proposition 3.14, $cl(\lambda)$ is a fuzzy resolvable set in (X, T). This implies that (X, T) is not a fuzzy hereditarily irresolvable space.

Proposition 3.18. *If* λ *is a fuzzy Baire set in a fuzzy almost P-space* (X, T), *then* $\lambda = \mu \wedge \eta$, *where* μ *is a fuzzy open set and* η *is a fuzzy somewhere dense set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Since (X, T) is a fuzzy almost *P*-space, by Theorem 2.20, the fuzzy residual set η is a fuzzy somewhere dense set in (X, T). Hence $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy somewhere dense set in (X, T).

4. Fuzzy Pseudo-open Sets in Fuzzy Topological Spaces

The purpose of this section is to study more deeply the notion of fuzzy pseudo-open sets in fuzzy topological spaces.

Definition 4.1 [21]**.** A fuzzy set λ in a fuzzy topological space (X, T) is called a *fuzzy pseudo-open set* if $\lambda = \mu \vee \gamma$, where μ is a fuzzy open set and γ is a fuzzy first category set in (X, T).

Example 4.1. Let $X = \{A, B, C\}$. Consider the following fuzzy sets $\lambda, \mu, \alpha, \beta, \theta$ and δ defined on *X*:

$\lambda : X \to I$ is defined by $\lambda(A) = 1$; $\lambda(B) = 0.2$; $\lambda(C) = 0.7$;

$\mu : X \to I$ is defined by $\mu(A) = 0.7$; $\mu(B) = 0.4$; $\mu(C) = 1$;

$\alpha : X \to I$ is defined by $\alpha(A) = 1$; $\alpha(B) = 0.8$; $\alpha(C) = 0.7$;

$\beta : X \to I$ is defined by $\beta(A) = 0.7$; $\beta(B) = 0.8$; $\beta(C) = 1$;

$\theta : X \to I$ is defined by $\theta(a) = 1$; $\theta(b) = 0.8$; $\theta(c) = 1$;

$\delta : X \to I$ is defined by $\delta(a) = 0.7$; $\delta(b) = 0.8$; $\delta(c) = 0.7$.

Then, $T = \{0, \lambda, \mu, \lambda \vee \mu, \lambda \wedge \mu, 1\}$ is a fuzzy topology on X. By computation, $cl(\lambda) = 1$; $cl(\mu) = 1$; $cl(\lambda \vee \mu) = 1$; $cl(\lambda \wedge \mu) = 1$ and $\text{int}(1 - \lambda) = 0$; $\text{int}(1 - \mu) = 0$; $\text{int}(1 - [\lambda \vee \mu]) = 0$; $\text{int}(1 - [\lambda \wedge \mu]) = 0$. Now, $\text{int}\, cl(1 - \lambda) = 0$; $\text{int}\, cl(1 - \mu) = 0$; $\text{int}\, cl(1 - [\lambda \vee \mu]) = 0$; $\text{int}\, cl(1 - [\lambda \wedge \mu]) = 0$. Thus fuzzy nowhere dense sets in (X, T) are $1 - \lambda$, $1 - \mu$, $1 - [\lambda \vee \mu]$ and $1 - [\lambda \wedge \mu]$. Further, $1 - [\lambda \wedge \mu] = (1 - \lambda) \vee (1 - \mu) \vee (1 - [\lambda \vee \mu]) \vee (1 - [\lambda \wedge \mu])$ and $1 - [\lambda \wedge \mu]$ is a fuzzy first category set in (X, T). Also, $\lambda \vee (1 - [\lambda \wedge \mu]) = \alpha$; $\mu \vee (1 - [\lambda \wedge \mu]) = \beta$; $(\lambda \vee \mu) \vee (1 - [\lambda \wedge \mu]) = \theta$ and $(\lambda \wedge \mu) \vee (1 - [\lambda \wedge \mu]) = \delta$. Hence α, β, θ and δ are fuzzy pseudo-open sets in (X, T). (It should be noted that fuzzy pseudo-open sets are not fuzzy open sets in (X, T).)

Proposition 4.1. *If* λ *is a fuzzy pseudo-open set in a fuzzy topological space* (X, T), *then there is a fuzzy pseudo-open set* θ *in* (X, T) *such that* $\lambda \leq \theta$.

Proof. Let λ be a fuzzy pseudo-open set in (X, T). Then, $\lambda = \mu \vee \gamma$, where μ is a fuzzy open set and γ is a fuzzy first category set in (X, T). By Theorem 2.16, there is a fuzzy first category set α in (X, T) such that $\gamma \leq \alpha \leq cl(\gamma)$. This implies that $\mu \vee \gamma \leq \mu \vee \alpha$ and then $\lambda \leq \mu \vee \alpha$, in (X, T). Let $\theta = \mu \vee \alpha$. Since μ is a fuzzy open set and α is a fuzzy first category set in (X, T), θ is a fuzzy pseudo-open set θ and $\lambda \leq \theta$, in (X, T).

The following proposition gives a condition under which fuzzy simply* open sets become fuzzy pseudo-open sets in fuzzy topological spaces.

Proposition 4.2. *If* λ *is a fuzzy simply* open set in a fuzzy topological space* (X, T) *in which fuzzy nowhere dense sets are* F_σ*-sets, then* λ *is a fuzzy pseudo-open set in* (X, T).

Proof. Let λ be a fuzzy simply* open set in (X, T). Then, $\lambda = \mu \vee \gamma$, where μ is a fuzzy open set and γ is a fuzzy nowhere dense set in (X, T). By hypothesis, the fuzzy nowhere dense set γ is a fuzzy F_σ-set and then $\gamma = \bigvee_{i=1}^{\infty}(\gamma_i)$, where (γ_i)'s are fuzzy closed sets in (X, T). Since γ is a fuzzy nowhere dense set, $\operatorname{int} cl(\gamma) = 0$ in (X, T). Since $\operatorname{int}(\gamma) \leq \operatorname{int} cl(\gamma)$, $\operatorname{int}(\gamma) = 0$, in (X, T). Now $\operatorname{int}(\gamma) = \operatorname{int}(\bigvee_{i=1}^{\infty}(\gamma_i)) \geq \bigvee_{i=1}^{\infty} \operatorname{int}(\gamma_i) = \bigvee_{i=1}^{\infty} \operatorname{int} cl(\gamma_i)$, which implies that $\bigvee_{i=1}^{\infty} \operatorname{int} cl(\gamma_i) = 0$ and then $\operatorname{int} cl(\gamma_i) = 0$ and thus (γ_i)'s are fuzzy nowhere dense sets in (X, T). Then $\gamma = \bigvee_{i=1}^{\infty}(\gamma_i)$, where (γ_i)'s are fuzzy nowhere dense sets in (X, T), implying that γ is a fuzzy first category set in (X, T). Thus, $\lambda = \mu \vee \gamma$, where μ is a fuzzy open set and γ is a fuzzy first category set in (X, T), implying that λ is a fuzzy pseudo-open set in (X, T).

The following proposition establishes that fuzzy pseudo-open sets in a fuzzy *D*-Baire topological space are fuzzy simply* open sets.

Proposition 4.3. *If* λ *is a fuzzy pseudo-open set in a fuzzy D-Baire space* (X, T), *then* λ *is a fuzzy simply* open set in* (X, T).

Proof. Let λ be a fuzzy pseudo-open set in (X, T). Then, $\lambda = \mu \vee \gamma$, where μ is a fuzzy open set and γ is a fuzzy first category set in (X, T). Since (X, T) is a fuzzy *D*-Baire space, the fuzzy first category set γ is a fuzzy nowhere dense set in (X, T). Thus, $\lambda = \mu \vee \gamma$, where $\mu \in T$ and γ

is a fuzzy nowhere dense set in (X, T). Hence λ is a fuzzy simply* open set in (X, T).

Remark 4.1. It is to be mentioned that fuzzy pseudo-open sets in fuzzy *D*-Baire topological spaces need not be fuzzy simply open sets.

Example 4.2. Let $X = \{A, B, C\}$. Consider the following fuzzy sets $\alpha, \beta, \delta, \gamma, \theta, \rho, \sigma, \eta$ and φ defined on *X*:

$\alpha : X \to I$ is defined by $\alpha(A) = 0.5$; $\alpha(B) = 0.3$; $\alpha(C) = 0.5$,

$\beta : X \to I$ is defined by $\beta(A) = 0.6$; $\beta(B) = 0.5$; $\beta(C) = 0.7$,

$\delta : X \to I$ is defined by $\delta(A) = 0.5$; $\delta(B) = 0.4$; $\delta(C) = 0.6$,

$\gamma : X \to I$ is defined by $\gamma(A) = 0.5$; $\gamma(B) = 0.6$; $\gamma(C) = 0.4$,

$\theta : X \to I$ is defined by $\theta(A) = 0.5$; $\theta(B) = 0.6$; $\theta(C) = 0.5$,

$\rho : X \to I$ is defined by $\rho(A) = 0.6$; $\rho(B) = 0.6$; $\rho(C) = 0.6$,

$\sigma : X \to I$ is defined by $\sigma(A) = 0.5$; $\sigma(B) = 0.6$; $\sigma(C) = 0.6$,

$\eta : X \to I$ is defined by $\eta(A) = 0.5$; $\eta(B) = 0.5$; $\eta(C) = 0.5$,

$\varphi : X \to I$ is defined by $\varphi(A) = 0.5$; $\varphi(B) = 0.5$; $\varphi(C) = 0.6$.

Then, $T = \{0, \alpha, \beta, \delta, 1\}$ is a fuzzy topology on *X*. By computation, $cl(\alpha) = 1 - \alpha$; $cl(\beta) = 1$; $cl(\delta) = 1$ and $\operatorname{int}(1 - \alpha) = \alpha$; $\operatorname{int}(1 - \beta) = 0$; $\operatorname{int}(1 - \delta) = 0$, in (X, T). Now $\operatorname{int} cl(1 - \alpha) = \alpha$; $\operatorname{int} cl(1 - \beta) = 0$ and $\operatorname{int} cl(1 - \delta) = 0$. Thus fuzzy nowhere dense sets in (X, T) are $1 - \beta$ and $1 - \delta$. Further, $\gamma = (1 - \beta) \vee (1 - \delta)$ and thus γ is a fuzzy first category set in (X, T). Also, $\operatorname{int} cl(\gamma) = \operatorname{int}(1 - \delta) = 0$. Thus, the fuzzy first category set γ is a fuzzy nowhere dense set in (X, T), which implies that (X, T) is a fuzzy *D*-Baire space.

By computation, $\alpha \vee \gamma = \theta$; $\beta \vee \gamma = \rho$; $\delta \vee \gamma = \sigma$ and thus θ, ρ and σ are the fuzzy pseudo-open sets in (X, T). Fuzzy simply* open sets in (X, T) are η, β, φ, θ, ρ and σ, where $\alpha \vee (1 - \beta) = \eta$; $\beta \vee (1 - \beta) = \beta$; $\delta \vee (1 - \beta) = \varphi$ and $\alpha \vee (1 - \delta) = \theta$; $\beta \vee (1 - \delta) = \rho$; $\delta \vee (1 - \delta) = \sigma$. It is observed that fuzzy pseudo-open sets θ, ρ and σ are fuzzy simply* open sets in (X, T). (It should be mentioned that fuzzy pseudo-open sets θ, ρ and σ are not fuzzy open sets in (X, T) and fuzzy simply* open sets η and φ are not fuzzy pseudo-open sets in (X, T).)

Also,

$$\operatorname{int} cl[bd(\theta)] = \operatorname{int} cl[cl(\theta) \wedge cl(1 - \theta)] = \operatorname{int} cl[(1 - \alpha) \wedge (1 - \alpha)]$$

$$= \operatorname{int} cl[(1 - \alpha)] = \operatorname{int}(1 - \alpha) = \alpha \neq 0$$

and θ is not a fuzzy simply open set in (X, T). Thus fuzzy pseudo-open set θ is not a fuzzy simply open set in fuzzy *D*-Baire space (X, T).

The following proposition establishes a condition under which fuzzy pseudo-open sets are fuzzy simply open sets.

Proposition 4.4. *If* λ *is a fuzzy pseudo-open set in a fuzzy hyperconnected fuzzy D-Baire space* (X, T), *then* λ *is a fuzzy simply open set in* (X, T).

Proof. Let λ be a fuzzy pseudo-open set in (X, T). Since (X, T) is a fuzzy *D*-Baire space, by Proposition 4.3, λ is a fuzzy simply* open set in (X, T). Also, since (X, T) is a fuzzy hyperconnected space, by Theorem 2.15, λ is a fuzzy simply open set in (X, T).

Remark 4.2. It is to be mentioned that fuzzy simply open sets in fuzzy hyperconnected fuzzy *D*-Baire spaces need not be fuzzy pseudo-open sets.

Example 4.3. Let $X = \{A, B, C\}$. Consider the following fuzzy sets α, β, γ and δ defined on *X*:

$\alpha : X \to I$ is defined by $\alpha(A) = 0.6$; $\alpha(B) = 0.8$; $\alpha(C) = 0.9$;

$\beta : X \to I$ is defined by $\beta(A) = 0.7$; $\beta(B) = 0.5$; $\beta(C) = 0.8$;

$\gamma : X \to I$ is defined by $\gamma(A) = 0.8$; $\gamma(B) = 1$; $\gamma(C) = 0.6$,

$\delta : X \to I$ is defined by $\delta(A) = 0.4$; $\delta(B) = 0.3$; $\delta(C) = 0.4$.

Then,

$$T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \beta \vee [\alpha \wedge \gamma], \alpha \wedge [\beta \vee \gamma], \gamma \wedge [\alpha \vee \beta], \alpha \wedge \beta \wedge \gamma, 1\}$$

is a fuzzy topology on *X*.

By computation, $cl(\alpha) = 1$; $cl(\beta) = 1$; $cl(\gamma) = 1$; $cl(\alpha \vee \beta) = 1$; $cl(\alpha \vee \gamma) = 1$; $cl(\beta \vee \gamma) = 1$; $cl(\alpha \wedge \beta) = 1$; $cl(\alpha \wedge \gamma) = 1$; $cl(\beta \wedge \gamma) = 1$; $cl(\beta \vee [\alpha \wedge \gamma]) = 1$; $cl(\alpha \wedge [\beta \vee \gamma]) = 1$; $cl(\gamma \wedge [\alpha \vee \beta]) = 1$ and $cl(\alpha \wedge \beta \wedge \gamma) = 1$ in (X, T). Since all the fuzzy open sets in (X, T) are fuzzy dense sets in (X, T), (X, T) is a fuzzy hyperconnected space.

Since all the fuzzy open sets are fuzzy dense, their complements $1 - \alpha$, $1 - \beta$, $1 - \gamma$, $1 - (\alpha \vee \beta)$, $1 - (\alpha \vee \gamma)$, $1 - (\beta \vee \gamma)$, $1 - (\alpha \wedge \beta)$, $1 - (\alpha \wedge \gamma)$, $1 - (\beta \wedge \gamma)$, $1 - (\beta \vee [\alpha \wedge \gamma])$, $1 - (\alpha \wedge [\beta \vee \gamma])$, $1 - (\gamma \wedge [\alpha \vee \beta])$ and $1 - (\alpha \wedge \beta \wedge \gamma)$ are fuzzy nowhere dense sets in (X, T). By computation, fuzzy first category sets in (X, T) are $1 - (\alpha \wedge \beta \wedge \gamma)$, $1 - (\beta \wedge \gamma)$ and $1 - \gamma$. Since each fuzzy first category set is a fuzzy nowhere dense set in (X, T), (X, T) is a fuzzy *D*-Baire space. Thus (X, T) is a fuzzy hyperconnected fuzzy *D*-Baire space.

By computation, fuzzy pseudo-open sets in (X, T) are all the fuzzy open sets and all the fuzzy simply*-open sets are all fuzzy open sets in (X, T). Now,

$$\begin{aligned} \operatorname{int} cl[bd(\delta)] &= \operatorname{int} cl[cl(\delta) \wedge cl(1-\delta)] = \operatorname{int} cl[\{1-(\alpha \wedge \beta \wedge \gamma)\} \wedge 1] \\ &= \operatorname{int} cl[1-(\alpha \wedge \beta \wedge \gamma)] = \operatorname{int}[1-(\alpha \wedge \beta \wedge \gamma)] = 0, \end{aligned}$$

and thus δ is a fuzzy simply open set in (X, T), but δ is not a fuzzy pseudo-open set in (X, T).

Proposition 4.5. *If* λ *is a fuzzy pseudo-open set in a fuzzy Baire fuzzy open hereditarily irresolvable space* (X, T), *then* λ *is a fuzzy simply* open set in* (X, T).

Proof. Let λ be a fuzzy pseudo-open set in (X, T). Then, $\lambda = \mu \vee \gamma$, where μ is a fuzzy open set and γ is a fuzzy first category set in (X, T). Since (X, T) is a fuzzy Baire fuzzy open hereditarily irresolvable space, by Theorem 2.14, (X, T) is a fuzzy *D*-Baire space and then the fuzzy first category set γ is a fuzzy nowhere dense set in (X, T). Thus, $\lambda = \mu \vee \gamma$, where $\mu \in T$ and γ is a fuzzy nowhere dense set in (X, T), implying that λ is a fuzzy simply* open set in (X, T).

Proposition 4.6. *If* λ *is a fuzzy pseudo-open set in a fuzzy Baire space in which each fuzzy first category set is a fuzzy closed set*, *then* λ *is a fuzzy simply* open set in* (X, T).

Proof. Let λ be a fuzzy pseudo-open set in (X, T). Then, $\lambda = \mu \vee \gamma$, where μ is a fuzzy open set and γ is a fuzzy first category set in (X, T). Because (X, T) is a fuzzy Baire space, by Theorem 2.9, $\operatorname{int}(\gamma) = 0$, in (X, T). By hypothesis, the fuzzy first category set γ is a fuzzy closed set and thus $cl(\gamma) = \gamma$, in (X, T). This implies that $\operatorname{int} cl(\gamma) = 0$. That is, γ is a fuzzy nowhere dense set in (X, T). Thus, $\lambda = \mu \vee \gamma$, where $\mu \in T$ and γ

is a fuzzy nowhere dense set in (X, T) implying that λ is a fuzzy simply* open set in (X, T).

Remark 4.3. In Example 4.2, for first nowhere dense sets $1 - \beta$ and $1 - \delta$, $\operatorname{int}([1 - \beta] \vee [1 - \delta]) = \operatorname{int}(\gamma) = 0$ and thus (X, T) is a Baire space.

Also, fuzzy first category set γ is a fuzzy closed set in (X, T) and fuzzy pseudo-open sets θ, ρ and σ are fuzzy simply* open sets in (X, T).

Proposition 4.7. *If* $\lambda = \mu \vee \gamma$, *where* μ *is a fuzzy open set and* γ *is a fuzzy* σ*-nowhere dense set in a fuzzy topological space* (X, T), *then* λ *is a fuzzy pseudo-open set in* (X, T).

Proof. Suppose that $\lambda = \mu \vee \gamma$, where μ is a fuzzy open set and γ is a fuzzy σ-nowhere dense set in (X, T). By Theorem 2.4, for the fuzzy σ-nowhere dense set γ, $1 - \gamma$ is a fuzzy residual set in (X, T) and thus γ is a fuzzy first category set in (X, T). Hence $\lambda = \mu \vee \gamma$, where μ is a fuzzy open set and γ is a fuzzy first category set in (X, T), implying that λ is a fuzzy pseudo-open set in (X, T).

5. Interrelations between Fuzzy Baire Sets and Fuzzy Pseudo-open Sets

The purpose of this section is to study the interrelations between fuzzy Baire sets and fuzzy pseudo-open sets in fuzzy topological spaces.

The following proposition gives a condition under which complements of fuzzy pseudo-open sets are fuzzy Baire sets.

Proposition 5.1. *If* λ *is a fuzzy pseudo-open set in a fuzzy topological space* (X, T) *in which each fuzzy closed set is a fuzzy open set, then* $1 - \lambda$ *is a fuzzy Baire set in* (X, T).

Proof. Let λ be a fuzzy pseudo-open set in (X, T). Then, $\lambda = \mu \vee \gamma$, where μ is a fuzzy open set and γ is a fuzzy first category set in (X, T). This implies that $1 - \lambda = 1 - (\mu \vee \gamma) = (1 - \mu) \wedge (1 - \gamma)$, in (X, T). Let $\alpha = 1 - \mu$ and $\eta = 1 - \gamma$. Then, α is a fuzzy closed set and η is a fuzzy residual set in (X, T). By hypothesis, each fuzzy closed set is a fuzzy open set in (X, T) and thus α is a fuzzy open set in (X, T). Hence $1 - \lambda = \alpha \wedge \eta$, where α is a fuzzy open set and η is a fuzzy residual set in (X, T), implying that $1 - \lambda$ is a fuzzy Baire set in (X, T).

The following proposition gives a condition under which complements of fuzzy Baire sets are fuzzy pseudo-open sets.

Proposition 5.2. *If* λ *is a fuzzy Baire set in* (X, T) *in which each fuzzy closed set is a fuzzy open set, then* $1 - \lambda$ *is a fuzzy pseudo-open set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). This implies that $1 - \lambda = 1 - (\mu \wedge \eta) = (1 - \mu) \vee (1 - \eta)$, in (X, T). Let $\alpha = 1 - \mu$ and $\gamma = 1 - \eta$. Then, α is a fuzzy closed set and γ is a fuzzy first category set in (X, T). By hypothesis, each fuzzy closed set is a fuzzy open set in (X, T) and thus α is a fuzzy open set in (X, T). Hence $1 - \lambda = \alpha \vee \gamma$, where α is a fuzzy open set and γ is a fuzzy first category set in (X, T), implying that $1 - \lambda$ is a fuzzy pseudo-open set in (X, T).

Proposition 5.3. *If* (λ_i)*'s* $(i = 1 \text{ to } \infty)$ *are fuzzy Baire sets in a fuzzy hyperconnected fuzzy second category (but not fuzzy Baire) space* (X, T), *then for a non-zero fuzzy open set* δ *in* (X, T), $\delta \vee [\vee_{i=1}^{\infty}(\lambda_i)]$ *is a fuzzy pseudo-open set in* (X, T).

Proof. Let (λ_i)'s $(i = 1 \text{ to } \infty)$ be fuzzy Baire sets in (X, T). Since (X, T) is a fuzzy hyperconnected fuzzy second category (but not fuzzy Baire) space, by Proposition 3.15, $\vee_{i=1}^{\infty}(\lambda_i)$ is a fuzzy first category set in (X, T). Then, for a non-zero fuzzy open set δ in (X, T), $\delta \vee [\vee_{i=1}^{\infty}(\lambda_i)]$ is a fuzzy pseudo-open set in (X, T).

Proposition 5.4. *If* λ *is a fuzzy Baire set in a fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then there exists a fuzzy pseudo-open set* δ *in* (X, T) *such that* $\delta \leq \text{int}(1 - \lambda)$.

Proof. Let λ be a fuzzy Baire set in (X, T). Then, $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in (X, T). Since η is a fuzzy residual set in (X, T), $cl(\eta) \neq 1$, in (X, T). (For otherwise, if $cl(\eta) = 1$ in (X, T), by Theorem 2.9, (X, T) would be a fuzzy Baire space.) Then, $1 - cl(\eta) \neq 0$, in (X, T). By Lemma 2.1, $\text{int}(1 - \eta) = 1 - cl(\eta) \neq 0$. Since $\eta \leq cl(\eta)$, by Theorem 2.11, $cl(\eta)$ is a fuzzy residual set in (X, T) and then $1 - cl(\eta)$ is a fuzzy first category set in (X, T). Thus $\text{int}(1 - \eta)$ is a fuzzy first category set in (X, T). Now $\lambda = \mu \wedge \eta$ implies that $1 - \lambda = 1 - (\mu \wedge \eta) = (1 - \mu) \vee (1 - \eta)$, in (X, T) and $\text{int}(1 - \lambda) = \text{int}[(1 - \mu) \vee (1 - \eta)] \geq \text{int}(1 - \mu) \vee \text{int}(1 - \eta)$. Let $\delta = \text{int}(1 - \mu) \vee \text{int}(1 - \eta)$. Since $\text{int}(1 - \mu)$ is a fuzzy open set and $\text{int}(1 - \eta)$ is a fuzzy first category set in (X, T), δ is a fuzzy pseudo-open set and $\text{int}(1 - \lambda) \geq \delta$, in (X, T).

Corollary 5.1. *If* λ *is a fuzzy Baire set in a fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then there exists a fuzzy pseudo-open set* δ *in* (X, T) *such that* $\lambda \leq 1 - \delta$.

Proof. Let λ be a fuzzy Baire set in (X, T). Since (X, T) is a fuzzy second category (but not fuzzy Baire) space, by Proposition 5.4, there exists a fuzzy pseudo-open set δ in (X, T) such that $\delta \leq \text{int}(1 - \lambda)$. By

Lemma 2.1, $\operatorname{int}(1-\lambda) = 1 - cl(\lambda)$ and thus $\delta \leq 1 - cl(\lambda)$. This implies that $cl(\lambda) \leq 1 - \delta$. Now $\lambda \leq cl(\lambda)$, which implies that $\lambda \leq 1 - \delta$, in (X, T).

Proposition 5.5. *If* λ *is a fuzzy Baire set in a fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then there exist a fuzzy pseudo-open set* δ *and a fuzzy* F_σ*-set* β *in* (X, T) *such that* $\delta \leq 1 - \lambda \leq \beta$.

Proof. Let λ be a fuzzy Baire set in (X, T). Since (X, T) is a fuzzy second category (but not fuzzy Baire) space, by Corollary 5.1, there exists a fuzzy pseudo-open set δ in (X, T) such that $\lambda \leq 1 - \delta$. Then, $\delta \leq 1 - \lambda$. By Corollary 3.1, for the fuzzy Baire set λ, there exists a fuzzy F_σ-set β in (X, T) such that $1 - \lambda \leq \beta$. This implies that $\delta \leq 1 - \lambda \leq \beta$, in (X, T).

Corollary 5.2. *If* λ *is a fuzzy Baire set in a fuzzy second category* (*but not fuzzy Baire*) *space* (X, T), *then there exist a fuzzy pseudo-open set* δ *and a fuzzy* G_δ*-set* α *in* (X, T) *such that* $\alpha \leq \lambda \leq 1 - \delta$.

Proof. Let λ be a fuzzy Baire set in (X, T). Since (X, T) is a fuzzy second category (but not fuzzy Baire) space, by Proposition 5.5, there exist a fuzzy pseudo-open set δ and a fuzzy F_σ-set β in (X, T) such that $\delta \leq 1 - \lambda \leq \beta$. Then, $1 - \delta \geq 1 - [1 - \lambda] \geq 1 - \beta$. Let $\alpha = 1 - \beta$. Thus α is a fuzzy G_δ-set and $\alpha \leq \lambda \leq 1 - \delta$, in (X, T).

Proposition 5.6. *If* λ *is a fuzzy Baire set in a fuzzy submaximal* (*fuzzy globally disconnected*) *fuzzy P-space* (X, T), *then for a fuzzy first category set* μ, $\lambda \vee \mu$ *is a fuzzy pseudo-open set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Since (X, T) is a fuzzy submaximal (fuzzy globally disconnected) space, by Proposition 3.9 (Proposition 3.10), λ is a fuzzy G_δ-set in (X, T). Also, since (X, T) is a fuzzy *P*-space, the fuzzy G_δ-set λ is a fuzzy open set in (X, T). Thus, for a fuzzy first category set μ in (X, T), $\lambda \vee \mu$ is a fuzzy pseudo-open set in (X, T).

Remark 5.1. In view of Proposition 5.6, wel have the following result: "If $\lambda = (\mu \wedge \eta) \vee \delta$, where μ is a fuzzy open set, η is a fuzzy residual set and δ is a fuzzy first category set in a fuzzy submaximal (fuzzy globally disconnected) fuzzy P-space (X, T), then λ is a fuzzy pseudo-open set in (X, T)".

Proposition 5.7. *If* λ *is a fuzzy Baire set in a fuzzy strongly hyperconnected space* (X, T), *then for a fuzzy first category set* μ *in* (X, T), $\lambda \vee \mu$ *is a fuzzy pseudo-open set in* (X, T).

Proof. Let λ be a fuzzy Baire set in (X, T). Since (X, T) is a fuzzy strongly hyperconnected space, by Proposition 3.3, λ is a fuzzy open set in (X, T). Thus, for a fuzzy first category set μ in (X, T), $\lambda \vee \mu$ is a fuzzy pseudo-open set in (X, T).

Remark 5.2. In view of Proposition 5.7, we have the following result: "If $\lambda = (\mu \wedge \eta) \vee \delta$, where μ is a fuzzy open set, η is a fuzzy residual set and δ is a fuzzy first category set in a fuzzy strongly hyperconnected space (X, T), then λ is a fuzzy pseudo-open set in (X, T)".

6. Conclusion

In this paper, several characterizations of fuzzy Baire sets and fuzzy pseudo-open sets in fuzzy topological spaces are established. The conditions under which fuzzy Baire sets become fuzzy pseudo-open sets are established. It is obtained that fuzzy Baire sets in fuzzy submaximal spaces and fuzzy globally disconnected spaces are fuzzy G_δ-sets and fuzzy Baire sets in fuzzy hyperconnected and fuzzy second category (but not fuzzy Baire) spaces are fuzzy nowhere dense sets. It is found that fuzzy Baire sets in fuzzy strongly hyperconnected spaces are fuzzy open sets and fuzzy Baire sets are obtained from fuzzy regular closed sets and fuzzy residual sets in fuzzy perfectly disconnected spaces and from fuzzy open sets and complements of fuzzy σ-nowhere dense sets in fuzzy topological spaces. It is established that fuzzy Baire sets in fuzzy *D*-Baire spaces possess fuzzy regular closed sets as their supersets and closures of fuzzy Baire sets in fuzzy hyperconnected and fuzzy second category (but not fuzzy Baire) spaces are fuzzy resolvable sets.

It is established that fuzzy pseudo-open sets in fuzzy *D*-Baire spaces are fuzzy simply* open sets and fuzzy pseudo-open sets in fuzzy hyperconnected fuzzy *D*-Baire spaces are fuzzy simply open sets. The conditions under which fuzzy simply* open sets become fuzzy pseudo-open sets in fuzzy topological spaces and fuzzy pseudo-open sets become fuzzy simply* open sets in fuzzy Baire spaces are established. It is obtained that fuzzy pseudo-open sets in a fuzzy Baire fuzzy open hereditarily irresolvable space are fuzzy simply* open sets. Conditions under which complements of fuzzy Baire sets become fuzzy pseudo-open sets and complements of fuzzy pseudo-open sets become fuzzy Baire sets are also obtained.

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