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PROOF OF THE "SLIDE AND DIVIDE" METHOD OF FACTORING

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Abstract

The "slide and divide" method of factoring quadratics $ax^2 + bx + c$, where $a \neq 1$ is gaining popularity in algebra courses. It has many video demonstrations online and the individuals creating these videos enjoy the method. They emphasize the simplicity of its arithmetic and its ability to remove guess work from the factoring process.

The purpose of this paper is not to demonstrate this technique, but to prove it rigorously. There are websites that challenge the mathematical authenticity of the technique, claiming it to be a "shuffling of cards" [2] and not mathematically sound. Although the comments in [2] are not very kind toward the method, it still begs the question as to why it works. I set out to rigorously prove this technique. To see if it is mathematically and pedagogically valid and sound.

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1. Introduction

The "slide and divide" method of factoring quadratics has gained tremendous popularity over the past few years. When I first saw the method demonstrated to me recently by Madeline, a student, I was extremely skeptical of the method. I informed the student this method has many errors when initially considering the logic behind it.

Looking at the process at face value, it appears a lot of mathematical logic is violated. Firstly, rewriting the quadratic $ax^2 + bx + c$ as $x^2 + bx + ac$ does not yield the same quadratic. Right here, we have two unequal expressions. How can the factorization be correct when the quadratics are different? Factorizations of polynomials are unique.

Secondly, when we divide only a part of the factor by the leading coefficient, this is also a violation. We emphasize that when working with expressions, to maintain equality from step to step, we may only add zero or multiply by one. We cannot simply divide parts of factors by values and expect the expressions to maintain the equality we are seeking.

Lastly, to rewrite factors as $ax + b$ instead of $x + \frac{b}{a}$ is highly dubious when working with expressions. You can get away with it in equations since you may multiply both sides by a and clear fractions. When working with expressions though, rewriting expressions from $ax + b$ to $x + \frac{b}{a}$ without the a factored out is incorrect as the expressions would not be equal.

When Madeline showed me this, I informed her that on the surface, I would not count this correct as the logic was not upheld. Her response was this method works and she had been using it for a while. I replied that if she could prove it or direct me to a site that proved it rigorously, I would accept it. She directed me to [2].

After seeing the page, and reading the comments on this page, I conducted my own search about this method. I saw several videos demonstrating the process, such as [1]. I was not able to find any rigorous proof of the method. In [3], the authors refer to the guess-and-check method or the AC-method. The comments stated in [2] and the lack of rigorous proof available to me inspired me to prove this method.

Consider the expression $ax^2 + bx + c$. The “slide and divide” method has three major steps:

(1) Starting with $ax^2 + bx + c$, change it to $x^2 + bx + ac$.

(2) Once you have $x^2 + bx + ac$ factored into $(x + m)(x + n)$, divide the m and n by the leading coefficient a. This gives $\left(x + \frac{m}{a}\right)\left(x + \frac{n}{a}\right)$.

(3) Reduce $\frac{m}{a} = \frac{p_1}{q_1}$ and $\frac{n}{a} = \frac{p_2}{q_2}$ and then rewrite the factors as

$$\left(x + \frac{m}{a}\right)\left(x + \frac{n}{a}\right) = (q_1x + p_1)(q_2x + p_2).$$

This document demonstrates a proof that this method is valid for factoring any quadratic expression. It looks like “magic” or a lot of hand waving, but it is not. The proof below exposes the “magic”.

Proof. Given the following quadratic expression $ax^2 + bx + c$, proceed as follows:

$$ax^2 + bx + c = \frac{(ax + m)(ax + n)}{a} \text{ multiply by 1 in the form } \frac{a}{a}$$

$$= \frac{1}{a}(a^2x^2 + anx + amx + mn) \text{ call this EQ (1)}$$

$$= \frac{a^2x^2 + ax(m + n) + mn}{a}$$

$$= ax^2 + (m + n)x + \frac{mn}{a}.$$

Now matching coefficients yield

$$\text{coefficients of } x^2 : a = a,$$

$$\text{coefficients of } x : m + n = b,$$

$$\text{constant terms: } \frac{mn}{a} = c.$$

When we solve this nonlinear system of equations, we have

$$m = b - n \Rightarrow \frac{(b-n)n}{a} = c \Rightarrow n(b-n) = ac.$$

Now we solve the equation for n:

$$n(b-n) = ac,$$

$$nb - n^2 = ac,$$

$$0 = n^2 - nb + ac.$$

The "slide and divide" method starts here! The first step is to change the quadratic to this one. This transition works because it is the resulting equation from the nonlinear system that is used to determine the solutions.

Use of the quadratic formula to solve this equation yields:

$$n = \frac{-(-b) \pm \sqrt{(-b)^2 - 4(1)(ac)}}{2(1)} = \frac{b \pm \sqrt{b^2 - 4ac}}{2}.$$

The next step in the process is to divide these solutions by the leading coefficient. This is legal because of EQ (1):

$$\frac{1}{a}(ax+m)(ax+n) = \frac{a}{a^2}(ax+m)(ax+n) \text{ multiply by 1 again in the form } \frac{a}{a}$$

$$= a\left(\frac{ax+m}{a}\right)\left(\frac{ax+n}{a}\right) \text{ distribute } 1/a \text{ on each factor}$$

$$= a\left(x + \frac{m}{a}\right)\left(x + \frac{n}{a}\right).$$

The final step in the "slide and divide" method is to reduce the fractions m/a and n/a and rewrite the factors without fractions involved. We use the extended factor theorem to support this.

Extended factor theorem. *Suppose that* $P(x)$ *is a polynomial of degree* $n \geq 1$. *Then the following are true*:

(i) *If* $x = \frac{p}{q}$ *is a solution to* $P(x)$, *then* $qx - p$ *is a factor of* $P(x)$.

(ii) *If* $qx - p$ *is a factor of* $P(x)$, *then* $x = \frac{p}{q}$ *is a solution to* $P(x)$.

If we reduce our fractions, then

$$\frac{m}{a} = \frac{p_1}{q_1} \quad \text{and} \quad \frac{n}{a} = \frac{p_2}{q_2}.$$

The extended factor theorem allows us to rewrite these factors as

$$a\left(x + \frac{m}{a}\right)\left(x + \frac{n}{a}\right) = a\left(x + \frac{p_1}{q_1}\right)\left(x + \frac{p_2}{q_2}\right) \text{ after reducing}$$

$$= a\left(\frac{q_1x + p_1}{q_1}\right)\left(\frac{q_2x + p_2}{q_2}\right)$$

$$= \frac{a}{q_1q_2}(q_1x + p_1)(q_2x + p_2).$$

This concludes the proof.

2. Demonstrations

In this section, we demonstrate the factorization process with expressions. When working with equations, the greatest common factor may be divided out. We have

$$\frac{a}{q_1q_2}(q_1x + p_1)(q_2x + p_2) = 0.$$

Since a, q_1, and $q_2 \neq 0$, we may divide both sides by the coefficient and we have

$$(q_1x + p_1)(q_2x + p_2) = 0 \Rightarrow x = -\frac{p_1}{q_1} \text{ or } x = -\frac{p_2}{q_2}.$$

In many demonstrations, I found online $a = q_1q_2$ for simplicity, but it is not required in general.

Example 1. Factor $8x^2 + 2x - 3$ by the "slide and divide" method.

Answer. Following our steps yields:

(1) Rewrite as $x^2 + 2x + (8)(-3) = x^2 + 2x - 24$.

(2) Factor this expression down:

$$x^2 + 2x - 24 = (x + 6)(x - 4).$$

Now divide the constants by the leading coefficient and reduce

$$(x + 6)(x - 4) = \left(x + \frac{6}{8}\right)\left(x - \frac{4}{8}\right) = \left(x + \frac{3}{4}\right)\left(x - \frac{1}{2}\right).$$

(3) Use the extended factor theorem to rewrite the factors. The leading coefficient is still a factor at this stage:

$$\begin{aligned} 8\left(x + \frac{3}{4}\right)\left(x - \frac{1}{2}\right) &= 8\left(\frac{4x + 3}{4}\right)\left(\frac{2x - 1}{2}\right) \\ &= \frac{8}{8}(4x + 3)(2x - 1) \\ &= (4x + 3)(2x - 1). \end{aligned}$$

Recalling the leading coefficient is still a factor in the third step is vital. Since we are not solving equations, we cannot simply remove the leading coefficient from the factorization.

Example 2. Use the "slide and divide" method to factor $15x^2 + \frac{15}{4}x - \frac{45}{8}$.

Answer. If we use the method directly, we have

(1) Rewrite the expression as $x^2 + \frac{15}{4}x - 15\left(\frac{45}{8}\right)$.

(2) Factor this expression

$$x^2 + \frac{15}{4}x - \frac{675}{8} = \left(x + \frac{45}{4}\right)\left(x - \frac{15}{2}\right).$$

Then divide the constants in each factor by the leading coefficient and reduce:

$$\left(x + \frac{\frac{45}{4}}{15}\right)\left(x - \frac{\frac{15}{2}}{15}\right) = \left(x + \frac{3}{4}\right)\left(x - \frac{1}{2}\right).$$

(3) Use the extended factor theorem to rewrite the expression:

$$\left(\frac{4x + 3}{4}\right)\left(\frac{2x - 1}{2}\right) = (4x + 3)(2x - 1).$$

Expanding this result leaves

$$(2x - 1)(4x + 3) = 8x^2 + 6x - 4x - 3 = 8x^2 + 2x - 3.$$

This is not our original expression. What happened? The leading coefficient was not considered in the last step. If we take care to remember that the last step includes the leading coefficient $\frac{a}{q_1 q_2}$, then it will be correct. Let us show this:

$$\begin{aligned}\frac{15}{(2)(4)}(2x - 1)(4x + 3) &= \frac{15}{8}(8x^2 + 2x - 3)\\ &= 15\left(x^2 + \frac{1}{4}x - \frac{3}{8}\right)\\ &= 15x^2 + \frac{15}{4}x - \frac{45}{8}\end{aligned}$$

and this does match our original expression. The coefficient $\frac{a}{q_1 q_2}$ is equal to the greatest common factor. It is always encouraged to factor the greatest common factor from expressions before starting any factoring procedure.

Example 3. Use the "slide and divide" method to factor $15x^2 + \frac{15}{4}x - \frac{45}{8}$.

Answer. We begin by factoring out the greatest common factor first:

$$15x^2 + \frac{15}{4}x - \frac{45}{8} = 15\left(\frac{1}{8}\right)(8x^2 + 2x - 3).$$

Applying the "slide and divide" method of factoring was done in Example 1.

3. Concluding Remarks

The important thing to notice in Examples 2 and 3 is that the leading coefficient must be considered throughout the factoring process. Many examples online do not demonstrate this. The examples are usually equations, so the leading coefficient is divided out. This may be misleading to students when learning to factor. Solving equations is not the same as factoring expressions.

I encourage individuals who demonstrate this process to add examples like Example 2. The leading coefficient is not required to be a multiple of the product of the coefficients of the variable in the individual factors. If one skips factoring out the greatest common factor initially, the "slide and divide" method is set up to factor out the greatest common factor eventually. Demonstrating this part of the technique further enhances the student's experience.

One may note that for students, it does seem like a way around rigorous mathematics. To counter this misconception, it may be helpful for instructors to incorporate an explanation of why the method works. Another course of action could be for instructors to guide students through this proof. With so many techniques available, and technology as well, it is vital for a student's education to include rigorous notions and proofs. Even if the surface demonstration seems easy, instructors know there is usually a lot of interesting mathematics behind such methods.

Now that I am familiar with this method and have proven that it does indeed work legitimately, I will incorporate this into my discussions and notes for students. I would like to extend my thanks to Madeline who demonstrated this technique to me. For me, this was truly a case of the student educating the teacher.

References

[1] BriThemathGuy (The Slide - n - Divide). https://youtu.be/o2t7MFt-PBk?si=2Bi3kU4BnrOe8uGz.

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