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ONSET OF CONVECTION OF A CASSON NANOFLUID WITH MAGNETIC EFFECT: A NUMERICAL STUDY

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Abstract

In this study, the start of convective instability in a Casson nanofluid under the influence of a transverse magnetic field is investigated using linear stability analysis. Because of their improved heat transfer properties and non-Newtonian rheology, Casson nanofluids have garnered a lot of interest in scientific and practical applications. The governing non-dimensional equations are solved using the normal

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modes method, which leads to an eigenvalue problem for the linear stability analysis. In MAT-LAB R2021a, the eigenvalue problem is solved with bvp4c. The physical observations have been made on parameters like the concentration Rayleigh number (Rn), the modified diffusivity ratio (Na), the Casson fluid parameter (β) and the Hartmann number (Ha). The findings show that the critical Rayleigh number is considerably lowered by raising the Casson fluid parameter and concentration Rayleigh number, which speeds up the beginning of convection. Additionally, it is noted that by enhancing solutal buoyancy effects, greater values of the modified diffusivity ratio contribute to the system's destabilization. On the other hand, by reducing flow disruptions brought on by the Lorentz force, the Hartmann number shows a substantial stabilizing effect. Additionally, the research reveals a complex interaction between magnetic effects and the Casson parameter, suggesting that in specific parameter regimes, non-Newtonian behavior can partially counteract magnetic stabilization. With possible ramifications for thermal management and process optimization in engineering applications, our findings provide fresh insights into managing convective instabilities in Casson nanofluids.

1. Introduction

Nowadays, non-Newtonian fluids are becoming more and more important in research and engineering, outweighing Newtonian fluids in terms of importance. Non-Newtonian fluids have been used in a wide range of fields, including drilling operations, the food processing industry, and metallurgical processes [1-7].

Interest has significantly increased recently, especially when it comes to nanofluids. Numerous uses in both the residential and industrial sectors are driving this increased interest [8]. During investigation of convective instability and heat transfer properties within nanofluids, Kim et al. [9] illustrated that augmenting the density and heat capacity of nanoparticles, coupled with a decrease in their thermal conductivity and shape factor, facilitates convective motion within a nanofluid system. Tzou [10-11] conducted a comprehensive examination of thermal instability issues in nanofluids, considering various boundaries through the application of eigenfunction series expansions. In a related study, Dhananjay et al. [12] investigated Rayleigh Benard convection in nanofluids with free-free boundaries using the Galerkin weighted method. Notably, they addressed the aspect of over stability, an aspect overlooked by Tzou [10-11].

The onset of convective phenomena in a nanofluid layer of finite depth was explored by Nield and Kuznetsov [13]. Their findings highlighted a significant variation in the threshold Rayleigh number, influenced by the distribution of nanoparticles. Same authors [14] investigated the initiation of convective phenomena within a layer of nanofluid [15-16]. Utilizing a one-term Galerkin weighted approximation, they established stability thresholds for both steady and oscillatory conditions.

The Casson model is currently used in the food business and plays a key role in the development of rheological models for human blood. Sugar, cocoa beans, and milk powder make up chocolate, which is subject to notable rheological alterations depending on its constituents and manufacturing process. Controlling the rheological behaviour is therefore essential in the production of chocolate. Additionally, the Casson model is essential for customising medications to the rheology of the human body [17-31].

Recently, Narayana Chary et al. [32] have studied thermal convection of a Casson nanofluid with Coriolis effect. Sharathkumar Reddy et al. [33] investigated thermal convection of a ferrofluid with the effect of helical force. Ramchandraiah et al. [34] have been studied linear and weakly nonlinear analyses of double diffusive convection in porous media with chemical reaction using LTNE model. Narayana Chary et al. [35] have investigated onset of magneto convective instability of a Casson nanofluid with helical force. Reddy et al. [36] have studied thermohaline convective of a Casson nanoflud in a porous layer. Reddy et al. [37] investigated onset of triply diffusive convection in a power law fluid saturated porous layer.

In the present paper, we study the linear analysis of a Casson nanofluid with magnetic effect for a range of necessary values of the physical parameters, the concentration Rayleigh number (Rn), the modified diffusivity ratio (Na), and the Casson fluid parameter (β). The paper is organized as follows: Section 2 discusses the governing equations, while Section 3 covers the linear theory. Finally, Sections 4 and 5 present the outcome of the results, followed by conclusion and discussion of the findings.

2. Mathematical Equations

The rheological equation for Casson fluid flow is

$$\tau_{ij} = \xi_B + \left(\frac{P_y}{\sqrt{2\pi}}\right)2E_{ij}, \; \pi > \pi_c, \tag{2.1}$$

$$\tau_{ij} = \xi_B + \left(\frac{P_y}{\sqrt{2\pi_c}}\right)2E_{ij}, \; \pi < \pi_c, \tag{2.2}$$

where

$$\begin{cases} P_y = \text{Yield stress}, \; \xi_B = \text{Dynamic viscosity}, \\ \pi_c = \text{Critical value of } \pi = E_{ij}E_{ij}, \\ \qquad (\text{where } E_{ij} \text{ is the } (i, j)\text{th segment of deforming rate}). \end{cases}$$

We now turn our attention to examining a layer of Casson nanofluid constrained between the planes defined by $z^* = 0$ and $z^* = d$, subject to a slight temperature gradient,

$$(T_0^* - T_1^*), \; T_0^* > T_1^*.$$

Consider a reference frame in which the z^*-axis is oriented vertically upward. The asterisk notation is employed to denote dimensional variables explicitly. The governing equations can be formulated as

Continuity of equation

$$\nabla^* \cdot v^* = 0. \tag{2.3}$$

Momentum of equation

$$\rho\left(\frac{\partial}{\partial t^*} + \mathbf{v}^* \cdot \nabla^*\right)v^* = -\nabla^* p^* + div\tau + [\phi^*\rho_p + (1 - \phi^*)(1 - \beta(T^* - T_0^*))]g$$

$$+ \sigma_1(\mathbf{V} \times B_0\hat{e}_z) \times B_0\hat{e}_z. \tag{2.4}$$

Using equation (2.3) in equation (2.4), we get

$$\rho_{f_0}\left(\frac{\partial v^*}{\partial t^*} + v^* \cdot \nabla^* v^*\right) = \left(1 + \frac{1}{\beta}\right)\mu\nabla^{*2}v^* - \nabla^* p^*$$

$$+ [\phi^*\rho_p + (1 - \phi^*)(1 - \beta(T^* - T_0^*))]g$$

$$+ \sigma_1(\mathbf{V} \times B_0\hat{e}_z) \times B_0\hat{e}_z. \tag{2.5}$$

In the absence of buoyancy terms along the x and y axes, the Boussinesq approximation was employed. Equation (2.5) delineates the inertial term on the left-hand side, while the right-hand side represents the strain gradient and the viscous time period.

The equation of nanoparticles changes when there is no chemical reaction,

$$\frac{\partial\phi^*}{\partial t^*} + v^* \cdot \nabla\phi^* = -\frac{1}{\rho_p}\nabla \cdot j_p. \tag{2.6}$$

Here diffusion mass flux of nanoparticles j_p is given as

$$j_p = j_{p,t}\, j_{p,b} = -\rho_p D_b\nabla\phi^* - \rho_p D_t\frac{\nabla T^*}{T_0^*}. \tag{2.7}$$

Combining equations (2.5) and (2.6), we obtain

$$\frac{\partial\phi^*}{\partial t^*} + v^* \cdot \nabla\phi^* = D_b\nabla^{*2}\phi^* + \left(\frac{D_t}{T_0^*}\right)\nabla^{*2}T^*. \tag{2.8}$$

Equation of energy

$$\left[(\rho c)_f \frac{\partial T^*}{\partial t^*} + (\rho c)_f v^* \cdot \nabla T^*\right]$$

$$= (\rho c)_p \left[D_b \nabla^* \phi^* \cdot \nabla^* T^* + \frac{D_t}{T_0^*} \nabla^* T^* \cdot \nabla^* T^*\right] + k \nabla^{*2} T^*. \tag{2.9}$$

Through the utilization of nano-outcomes such as Brownian diffusion and thermophoresis, equation (2.8) achieves a balance between convection and conduction terms amidst the existence of nanoparticles and an internal heat source,

$$\begin{cases} v^* = \text{Velocity},\ p^* = \text{Pressure},\ t^* = \text{Time}, \\ \beta = \mu_B \dfrac{\sqrt{2\pi}}{P_y} = \text{The Casson fluid parameter}, \\ \mu = \text{Viscosity},\ g = \text{Gravity}, \\ k = \text{Effective thermal conductivity},\ \rho = \text{The density}, \\ D_b = \text{Diffusion coefficient of Brownian}, \\ D_t = \text{Thermophoresis diffusion coefficient}, \\ T^* = \text{The temperature},\ \rho c = \text{The heat capacity}, \\ \phi^* = \text{The particle volume fraction}. \end{cases}$$

Assuming that the nanoparticles' volumetric fraction and temperature are constant at the initial stage. We consider the stress-free boundary conditions. In hydrodynamic stability problems, stress-free conditions allow for more unstable modes than no-slip conditions, as they reduce the damping effect of viscosity,

$$Z^* = 0;\ T^* = T_0^*,\ \phi^* = \phi_0^*, \tag{2.10}$$

$$Z^* = d;\ T^* = T_1^*,\ \phi^* = \phi_1^*. \tag{2.11}$$

At the basic flow, we anticipate that the fluid layer is relaxed and the fraction of nanoparticles is constant, while the other variables primarily fluctuate along the horizontal axis.

At the conduction state the Casson nanofluid is assumed to be at rest, and hence, the temperature varies in the z-direction only and the solutions of the same are given by

$$v_b = 0,\ \phi_b = 0,\ T_b = T_0 - \left(\frac{\Delta T}{d}\right) z. \tag{2.12}$$

The non-dimensional parameters are

$$\begin{cases} (x,\ y,\ z) = \left(\dfrac{x^*}{d},\ \dfrac{y^*}{d},\ \dfrac{z^*}{d}\right); & t = \dfrac{T^* \alpha_f}{d2}; \\ (u,\ v,\ w) = \left(\dfrac{u^* d}{\alpha_f},\ \dfrac{v^* d}{\alpha_f},\ \dfrac{w^* d}{\alpha_f}\right); & \alpha_f = \dfrac{k}{(\rho c_p)_f}; \\ \phi = \dfrac{(\phi^* - \phi_0^*)}{(\phi_1^* - \phi_0^*)}; & T = \dfrac{(T^* - T_1^*)}{(T_0^* - T_1^*)}; \\ p = \dfrac{p^* K2}{\mu \alpha_f}. & \end{cases}$$

So far, it has been assumed that the spatial variations of k and μ are negligible. Indicates "p", "f" and "0" seek advice from particles, fluids, or reference variables. Then equations (2.3), (2.5), (2.8) and (2.9) reduce

$$\nabla \cdot v = 0, \tag{2.13}$$

$$\frac{1}{\Pr}\frac{\partial v}{\partial t} + \frac{1}{\Pr} v \cdot \nabla v = -\nabla p + \left(1 + \frac{1}{\beta}\right)\nabla^2 v - Rm e_z + RaT e_z - Rn\phi e_z$$

$$+ Ha^2[(\mathbf{V}' \times \hat{e}_z) \times \hat{e}_z], \tag{2.14}$$

$$\frac{\partial T}{\partial t} + v \cdot \nabla T = w + \nabla^2 T + \frac{NaNb}{Le}\nabla T \cdot \nabla T + \frac{Nb}{Le}\nabla\phi \cdot \nabla T, \tag{2.15}$$

$$\frac{\partial \phi}{\partial t} + v \cdot \nabla \phi = \frac{Na}{Le}\nabla^2 T + \frac{1}{Le}\nabla^2 \phi, \tag{2.16}$$

where

$$\begin{cases} Ra = \rho_{f0} g K^3 \beta_1 \dfrac{(T_0^* - T_1^*)}{\mu \alpha_f} (\text{Rayleigh number}), \\ Le = \dfrac{\alpha_f}{D_b} (\text{Lewis number}), \\ \Pr = \dfrac{\mu}{\rho \alpha_f} (\text{Prandtl number}),\ Ha = B_0 L \sqrt{\dfrac{\sigma_1}{\mu}} (\text{Hartmann number}), \\ Nb = (\rho c)_p \dfrac{(\phi_0^* - \phi_1^*)}{\rho c} (\text{Particle density increment}), \\ Na = \dfrac{D_t}{D_b} \dfrac{(T_0^* - T_1^*)}{T_1(\phi_1^* - \phi_0^*)} \\ \quad (\text{Adjusted diffusivity proportion(modified diffusivity ratio)}), \\ Rm = \left(\dfrac{\rho_p \phi_0^* + \rho_{f0}(1 - \phi_0^*)}{\mu \alpha_f} \right) g K^3 (\text{Thermal Rayleigh number}), \\ Rn = \dfrac{(\rho_p - \rho_{f0})(\phi_1^* - \phi_0^*)}{\mu \alpha_f} g K^3 (\text{Nanoparticle Rayleigh number}). \end{cases}$$

3. Linear Stability Analysis

To explore linear theory, consider the linear part of equations (2.13)-(2.16),

$$\frac{1}{\Pr}\frac{\partial v}{\partial t} = -\nabla p + \left(1 + \frac{1}{\beta}\right)\nabla^2 v - Rme_z + RaTe_z - Rn\phi e_z$$

$$+ Ha^2[(\mathbf{V}' \times \hat{e}_z) \times \hat{e}_z], \tag{3.1}$$

$$\frac{\partial T}{\partial t} = w + \nabla^2 T + \frac{Nb}{Le}\nabla\phi \cdot \nabla T + \frac{NaNb}{Le}\nabla T \cdot \nabla T, \tag{3.2}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Le}\nabla^2 \phi + \frac{Na}{Le}\nabla^2 T. \tag{3.3}$$

In order to remove the pressure term in momentum equation, we take the third components of curl of equation (3.1) and curl of curl of equation (3.1), and obtain

$$\left(\frac{1}{\Pr}\frac{\partial}{\partial t} - \left(1 + \frac{1}{\beta}\right)\nabla^2 + Ha^2\right)w_z = 0, \tag{3.4}$$

$$\left(\frac{1}{\Pr}\frac{\partial}{\partial t} - \left(1 + \frac{1}{\beta}\right)\nabla^2 + Ha^2\frac{\partial^2}{\partial z^2}\right)\nabla^2 w + (Rn\phi - RaT)\nabla_h^2 = 0, \tag{3.5}$$

where

$$\omega_z = (\nabla \times \mathbf{V}) \cdot \hat{e}_z,\ \nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},\ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

We convert the above system by putting the normal mode solution as $(w, T, \phi) = [W(z), \theta(z), \Phi(z)]e^{i(lx+my)+i\omega t}$ into equations (3.2), (3.3) and (3.5),

$$\left(\frac{1}{\Pr}(D^2 - q^2)i\omega - \left(1 + \frac{1}{\beta}\right)(D^2 - q^2)^2 + Ha^2D^2\right)w$$
$$- (Rn\phi - RaT)q^2 = 0, \tag{3.6}$$

$$\left(i\omega - (D^2 - q^2) - \frac{Nb}{Le}D + 2\frac{NaNb}{Le}D\right)\theta - W + D\Phi = 0, \tag{3.7}$$

$$\left(i\omega - \frac{1}{Le}(D^2 - q^2)\right)\Phi - \frac{Nb}{Le}(D^2 - q^2)\theta = 0, \tag{3.8}$$

$$z = 0, 1 : W = DW = \theta = \Phi = 0. \tag{3.9}$$

It is assumed that principle of exchange of instabilities holds. Substituting $\omega = 0$ into above equations,

$$\left(-\left(1 + \frac{1}{\beta}\right)(D^2 - q^2)^2 + Ha^2D^2\right)w - (Rn\phi - RaT)q^2 = 0, \tag{3.10}$$

$$\left(-(D^2 - q^2) - \frac{Nb}{Le}D + 2\frac{NaNb}{Le}D\right)\theta - W + D\Phi = 0, \tag{3.11}$$

$$\left(-\frac{1}{Le}(D^2 - q^2)\right)\Phi - \frac{Nb}{Le}(D^2 - q^2)\theta = 0, \tag{3.12}$$

$$z = 0, 1 : W = DW = \theta = \Phi = 0. \tag{3.13}$$

Now the above eigenvalue problem is solved for following boundary conditions.

Free-free boundary conditions

$$z = 0, 1 : W = D^2W = \theta = \Phi = 0. \tag{3.14}$$

Rigid-rigid boundary conditions

$$z = 0, 1 : W = DW = \theta = \Phi = 0. \tag{3.15}$$

The above eigenvalue problem is solved by using bvp4c routine in MATLAB [36-37].

4. Results in Discussion

The influence of magnetic field on the convective instability of Casson nanofluid is examined. As per our knowledge, the present problem has not been studied before. The non-dimension governing parameters of the onset of the convection are the Rayleigh number (Ra), the Casson fluid parameter (β), Hartmann number (Ha^2), modified diffusivity ratio (Na), concentration Rayleigh number (Rn). The eigenvalue problem for linear stability analysis is solved by employing bvp4c routine in Matlab R2021a. We assume that the principle of exchange of stabilities holds. Hence, the neutral stability condition happens with stationary modes. We consider the parameters range as $0 \leq Ha^2 \leq 10, 0 \leq Rn \leq 10, 0 \leq Na \leq 1$ and $0 \leq \beta \leq 10$.

To validate the methodology used in present analysis, the results obtained from this methodology are compared with the results published by Chandrasekhar [1] under limiting case. For a Newtonian fluid, in the absence of magnetic field, the present problem can be reduced to Chandrasekhar [1]. Table 1 shows the very good agreement of our bvp4c results with the critical Rayleigh number, Ra_c, and critical wave number, a_c, given in Chandrasekhar [1].

Table 1. Comparison table

Nature of boundaries	Chandrasekhar [1]		Present study	
	Ra_c	a_c	Ra_c	a_c
Free-free	657.511	2.221	657.511	2.221
Rigid-rigid	1707.762	3.117	1707.76182	3.11636

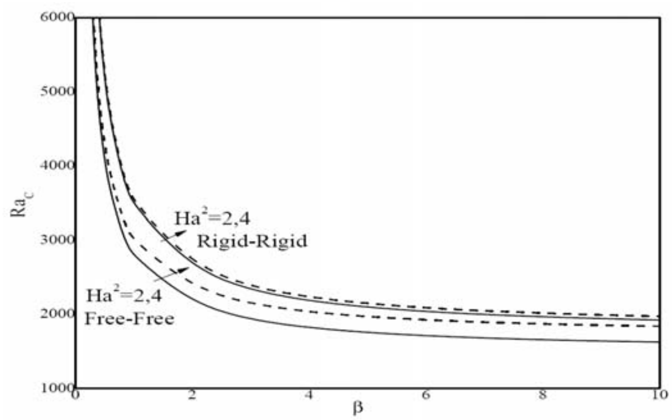

Figure 1. Critical Ra versus β for fixed values of $Rn = 5$ and $Na = 0.5$ for different values Ha^2.

Figure 1 illustrates the impact of the Casson fluid parameter for both numerically and analytically. This figure shows that, across a range of Ha^2 values, the critical Ra is a monotonically decreasing function of β, indicating that the Casson parameter destabilizes the flow. The yield stress of the fluid increases with Casson parameter, which leads to a decrease in the amplitude of the oscillations and leads to the destabilizing.

The relationship between Ra and Na for the fixed values of all parameters is depicted in Figure 2 for both numerically and analytically. The findings show a consistent pattern where Ra falls as Na rises across all Ha^2 values, indicating that Na has a destabilizing effect on the system. An increase in volumetric fraction increases the Brownian motion of the nanoparticles, which causes the destabilizing effect. Furthermore, as modified diffusivity ratio *Na* increases, the temperature difference also increases, which shows the destabilized effect of modified diffusivity ratio.

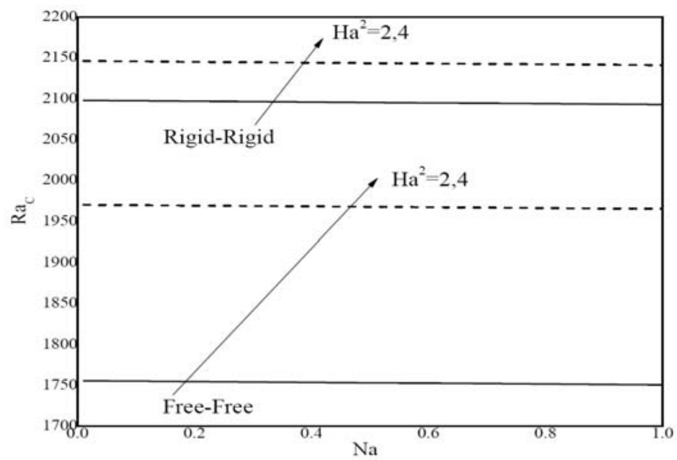

Figure 2. Critical *Ra* versus *Na* for fixed values of $Rn = 5$ and $\beta = 5$ for different values Ha^2.

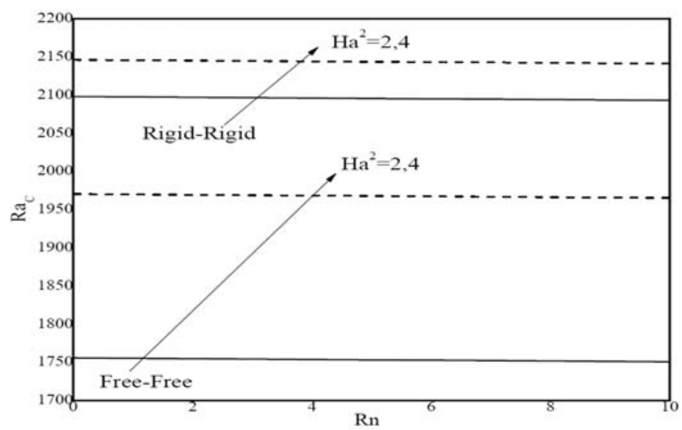

Figure 3. Plot of critical value of *Ra* versus *Rn* for fixed values of $Na = 0.5$ and $\beta = 5$ for different values Ha^2.

Figure 3 displays the variation of critical *Ra* with *Rn* for both numerically and analytically. This image makes it evident that for all Ha^2 values, *Ra* falls as *Rn* rises. This observation clearly indicates that *Rn* exerts a destabilizing influence on the system. Furthermore, it is observed that the stability of the system follows a monotonic pattern for any boundary condition.

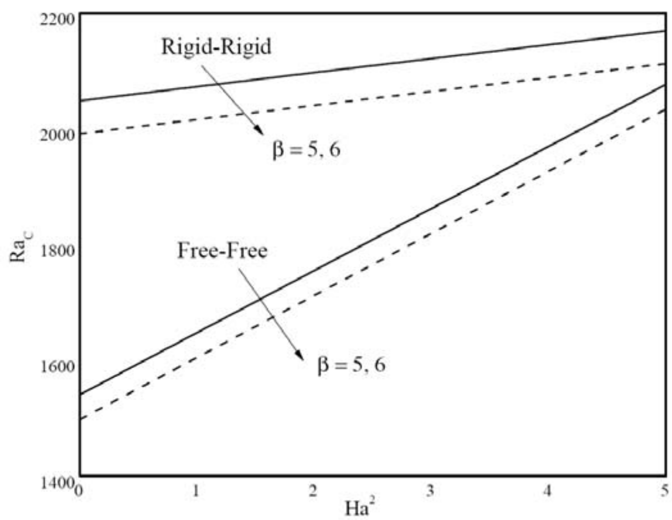

Figure 4. Plot of critical value of *Ra* versus Ha^2 for fixed values of $Rn = 1$ and $Na = 0.3$ for different values β.

Figure 4 shows how Ha^2 effects the system for stationary modes for both numerically and analytically. Figure 4 shows that, across a range of β values, the critical *Ra* rises as Ha^2 does. Consequently, a rise in Ha^2 raises the stability of the system. Magneto-convection plays a crucial role in regulating heat transfer within a system. The magnetic field generates the Lorentz force, which influences fluid motion. When this force is weaker than viscous or turbulent pressure, convective motions twist and stretch the magnetic field, intensifying turbulence. Conversely, if the Lorentz force surpasses viscous or turbulent pressure forces, the magnetic field directs plasma movement along its lines, effectively suppressing convection.

And also, from all figures it can be seen that for rigid-rigid boundaries system is more stable and less stable in the case of free-free boundaries.

5. Conclusions

The problem of convective instability of a Casson nanofluid with a magnetic effect is investigated in this paper. The resulting eigenvalue problem is solved numerically with bvp4c. The present results agree with previous findings. The following observations were made employing various physical parameters.

- The system becomes destabilized due to the concentration Rayleigh number.
- The system is destabilized by the modified diffusivity ratio parameter.
- The system instability is caused by the Casson fluid parameter.
- The flow is stabilized by the Hartmann number. It can be seen that for rigid-rigid boundaries system is more stable and less stable in the case of free-free boundaries.

We hope to expand our research in the future by investigating non-linear analysis of Casson fluid with various external fields.

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