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BURNING NUMBER OF SNAKE RELATED GRAPHS

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Abstract

Graph burning refers to the minimum number of moves necessary for data propagation in a network. Data circulation is modeled as a graph, and the concept of graph burning is used. Determining the network's burning number can reveal how quickly information spreads among followers and how it impacts the entire network. Even when memes are shared on social media sites like Facebook and Instagram, the network's burning number may be used to gauge how quickly information reaches its followers and how much impact it has on the entire network. In this paper, we have found the burning number for certain graphs, like the web graph and some snake-related graphs, which include the triangular snake graph, the quadrilateral snake graph, and the pentagonal snake graph.

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1. Introduction

One of the essential aspects for humans to thrive is communication. Communication facilitates the exchange of ideas and the development of relationships. The number of communication platforms is increasing more quickly in the modern world. At times, decision-making is also influenced by the information that is communicated. Information sharing connects people all over the world with both positive and negative impacts. A huge increase in the use of social media platforms has led to global connections, enabling effective data sharing among people for personal communication and communication in business. It also has a radical effect on the power of the government's elective leaders.

A graph can be modeled to represent the influencer's relationship with the individuals and the link that is created among them. The process of burning a graph helps to examine the network's characteristic information distribution. The graph which is considered in this paper is finite, simple and undirected. Choosing one unburned node is the first stage in the graph burning process. A fresh source is selected and the neighbours of the burned vertex are permitted to burn in the following steps. Until every vertex in the network is burned, the process is repeated. $b(G)$ is used to denote the burning number of the graph and it is defined as the minimum number of moves taken to burn the entire graph. A burning sequence is defined by Bonato et al. [1] as follows: for a simple graph G with V vertices and E edges, a burning sequence of G is defined as $(k_1, k_2, k_3, ..., k_n)$ a sequence of vertices of G such that for $l \leq i \leq n,$ the fire spread from k_i will burn only all the nodes within distance n_i from k_i by the end of the nth step. On the other hand, every node $v \in V(G)$ must be either a source of fire or burned from atleast one of the sources of fire by the end of the nth step. Burning number of a graph G is denoted by $b(G).$ The minimum $b(G)$ relates the swift spread in G.

2. Literature Review

The idea of graph burning was brought in by Bonato et al. [1]. This novel model was used to determine the burning number including upper and lower boundaries by Song et al. [3]. Optimal solutions for graph burning were given in order to assess the effectiveness of non-exact algorithms by Garcia-Diaz et al. [2]. For the generalized burning number of graphs, Li et al. [10] provided a new parameter. Based on the centrality measure of the eigenvector, Gautam et al. [7] offered three strategies. An attempt was made to apply the heuristic for disconnected graphs since it was important to order the component in these graphs. The benefits of the methods were put into practice, and these heuristics were tested for vertices with over 50,000 nodes. In their study of parameterized complexity, Kobayashi and Otachi [9] employed a distinct method to demonstrate parameterization by distance to split graphs. By choosing activators, Sumathi and Hannah Grace [8] suggested an algorithm to determine the burning number. Also, by using this approach, procedure for determining the burning number was improved. By creating methodical equipment to serve as test beds, Omar and Rohilla [5] described how to solve conjectures for a number of graph classes. In order to obtain the burning number for graphs with a single chord, Torun and Akyar [6] calculated the burning number for Jahangir graphs. Li [11] described the burning number for graphs with a value of three.

3. Main Results

3.1. Burning number of a web graph

Definition 3.1. The *pendant vertices* of the helm are joined to obtain a cycle and then by adding to each vertex of the outer cycle, a single pendant edge web graph Wb_n is formed [4]. Figure 1 shows the vertex representation.

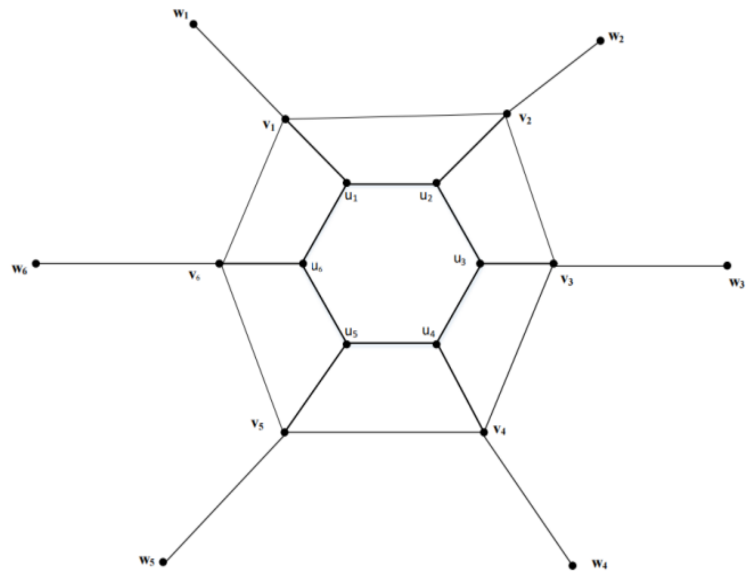

Figure 1. Web graph Wb_6.

Theorem 3.1. *The burning number of the web graph* Wb_n *is* $\lceil \sqrt{n} \rceil + 1$.

Proof. Let the vertices of the web graph be labeled as follows: The vertices of the inner cycle be labeled as u_i, the vertices of the outer cycle be labeled as v_j and the vertices that are formed by attaching a pendant edge to each vertex of the outer cycle be labeled as w_k, for $1 \leq i, j, k \leq n$. Let us assume that Wb_n is burnt in $\lceil \sqrt{n} \rceil$ steps by taking the first initiator from v_i. If the vertex which is burnt in step $\lceil \sqrt{n} \rceil$ is v_x, then we still find the existence of unburnt vertices that needs to be burnt in one more step. Hence $\lceil \sqrt{n} \rceil$ cannot be the burning number.

We use the following procedure to explain the chosen activator and demonstrate the vertices that are being burnt. This is done with a help of a step-by-step method. In each step, the vertices that are being chosen and the vertices that get burned are explained. Also, the vertices that are yet to be burned are mentioned.

In step 1, the process of burning starts by burning v_3 as the first activator. Hence it activates the neighbouring vertices v_2, v_4, u_3 and w_3 to be burned in step 2. To complete step 2, a source has to be selected, let it be v_5. In the next step 3, v_1, u_2, u_4, u_5, w_2, w_4, w_5 get burned. We find that w_1 and u_1 are the vertices that are yet to be burned. If w_1 is selected as the source to complete step 3, then u_1 is burnt in step 4. Thus, for $n > 5$, the first activator is selected as $v_{\sqrt{n}}$, this burns the neighbouring vertices. The procedure of burning gets iterated. We find all the vertices being burnt in step $\lceil \sqrt{n} \rceil + 1$.

3.2. Burning number of snake graphs

Definition 3.2.1. The *triangular snake graph* is formed from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to new vertex v_i for $i = 1, 2, ..., n-1$ and $|V(G)| = 2n - 1$ [4]. The vertex representation is given in Figure 2.

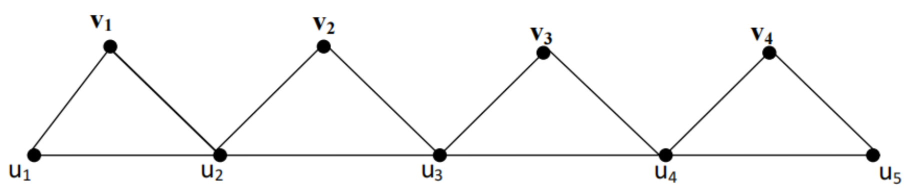

Figure 2. Triangular snake graph TS_5.

Theorem 3.2.1. *The burning number of a triangular snake graph* TS_n, $n \geq 3$ *is* $\left\lfloor \frac{-1 + 4\sqrt{(n-2)} + 1}{2} \right\rfloor + 2.$

Proof. The selected activator will be explained and the vertices that are being burned will be showed using the following process. A methodical approach is used to accomplish this. The vertices that are selected and those that are burned are described in each step.

The procedure of a graph burning involves burning u_i for $1 \leq i \leq n$.

Let the first activator be $u\sqrt{n}$ which gets burned at step 1. The fire gets transmitted to the neighbouring vertices which results in the burning of $u_{\sqrt{n}-1}$, $u_{\sqrt{n}+1}$ and the corresponding v_i for $1 \le i \le n-1$ at step 2. By selecting an unburnt u_i for $1 \le i \le n-1$, step 2 ends. The process is carried out until all the nodes are burnt.

Without loss of generality, let us consider the case where $n = 5$.

Let the node, which is set to propagate fire, be u_3 in step 1, which leads u_2, u_4, v_2, v_3 to be burnt in step 2. Step 2 gets completed by choosing u_5 as the second propagator. Hence u_1, v_1 and v_4 get burned in step 3. The process gets iterated by selecting unburnt vertex in each step. Thus, for TS_n,

$$b(G) = \left\lfloor \frac{-1 + 4\sqrt{(n-2)} + 1}{2} \right\rfloor + 2.$$

Definition 3.2.2. The *quadrilateral snake graph* is formed from a path $x_1, x_2, ..., x_n$ by connecting x_i and x_{i+1} to two new nodes y_i and z_i and then connecting y_i and z_i, for $i = 1, 2, ..., n-1$ [4]. Figure 3 shows the vertex representation of the QS_4.

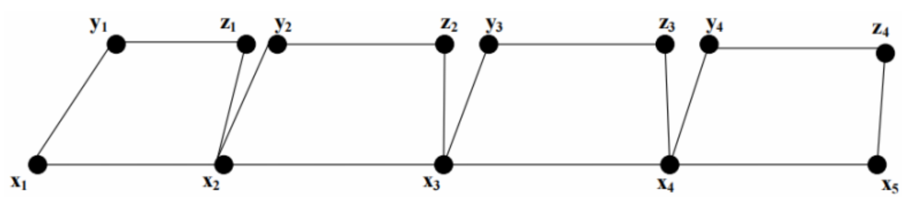

Figure 3. Quadrilateral snake graph QS_4.

Theorem 3.2.2. *The burning number of a quadrilateral snake graph* QS_n *is*

$$\begin{cases} \lceil \sqrt{n} \rceil & \text{if } n = k^2 + 1, \\ \lceil \sqrt{n} + 1 \rceil & \text{otherwise.} \end{cases}$$

Proof. Let the first node that is set to propagate be $x_{\sqrt{n}}$. $N(x_{\sqrt{n}})$ gets burned which includes $x_{\sqrt{n}} + 1$, $x_{\sqrt{n}} - 1$, and the corresponding y_i, z_i. There exist two adjacent nodes of x_i, $i = 2 \cdots n - 1$ getting burnt exactly one less than that of $\left\lfloor \frac{-2 + 4\sqrt{(n-3)} + 4}{2} \right\rfloor + 3$ so that y_i and z_i are burnt in the next step. The procedure is set to repeat until all the vertices are burnt. We can also state that for $n = k^2 + 1$, there exists exactly one vertex left to be taken as source in step $\lceil \sqrt{n} \rceil$. Hence $b(G) = \lceil \sqrt{n} \rceil$, otherwise $\lceil \sqrt{n} \rceil + 1$. Let us consider the case for $n = 4$ and $n = 5$ which is of the form $k^2 + 1$. In both the cases, the burning number is 3 since for $n = 5$, y_1 is the only vertex, which is unburnt, so that it can be taken as source 3 to complete that step. Hence the proof.

Definition 3.2.3. The *pentagonal snake graph* is formed from a path $x_1, x_2, ..., x_n$ by replacing every edge of the path by a cycle C_5 [4]. Figure 4 shows the vertex representation.

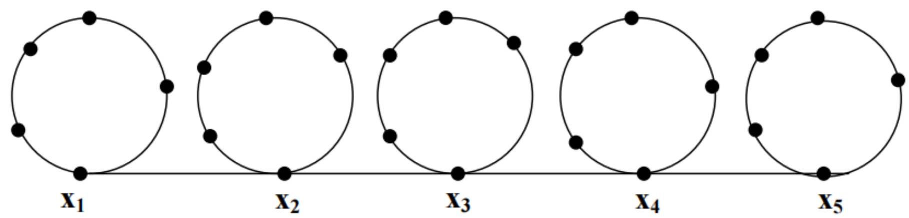

Figure 4. Pentagonal snake graph PS_5.

Theorem 3.2.3. *The burning number of a pentagonal snake graph* PS_n *is* $\left\lfloor \frac{-3 + 4\sqrt{(n-6)} + 9}{2} \right\rfloor + 3.$

Proof. To illustrate the vertices that are being burned and to explain the selected activator, we follow the below mentioned steps. A methodical approach is used to accomplish this. The vertices that are selected and the vertices that are burnt are described in each step.

The burning of pentagonal snake graph is initiated by first selecting an activator from the set of vertices of path P_n, let it be $\lceil \sqrt{n} \rceil$ at step 1. Burning the neighbouring vertices in step 2 and by repeating the procedure, all the nodes of the graph are burnt. In the process of burning, we note that two adjacent nodes of the path, namely, x_i and x_{i+1} are burnt in step $\left\lfloor \frac{-3+4\sqrt{(n-6)}+9}{2} \right\rfloor + 3$ and the corresponding vertex of the cycle, namely, c_1 is selected as the final source. The node c_1 that is attached to x_1 and x_2 is chosen as source in step $\left\lfloor \frac{-3+4\sqrt{(n-6)}+9}{2} \right\rfloor$.

4. Conclusion

More people are impacted by data transmission in social networks. In certain cases, the information shared on social media even influences people's decisions. This study enables us to know how a specific piece of data propagates over a network in the quickest time. Because it helps identify the bare minimal stages in which the spread has taken place, the process of graph burning enables us to ascertain how quickly a network can be affected. The challenge of circulating misinformation throughout the network causes chaos and occasionally mistrust. The node that has a minimum connectivity can be chosen in subsequent steps such that maximum $b(G)$ is obtained. Thus, the information that is spread does not get transferred rapidly in a network. In real-world networks, the decision to calculate the burning number for snake-related graphs works well.

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