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PURE AXIAL SHEAR OF A PSEUDO-ELASTIC LONG CIRCULAR CYLINDRICAL TUBE

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Abstract

We apply the theory of pseudo-elasticity developed by Ogden and Roxburgh to a problem related to non-homogeneous deformation, namely pure axial shear of a pseudo-elastic long circular cylindrical tube. Loading, affected by application of a specified rotation of the outer surface of the tube relative to the inner one, is described by an isotropic elastic strain-energy function. Here we have shown that if the maximum applied shear stress is below a certain critical value, then there is no residual strain after the shearing stress is removed, while if it is greater than a second critical value, then there is residual strain throughout the tube. In this paper, residual strain is calculated explicitly in context of a particular material model. The stress softening effect of the shear stress on unloading is numerically compared with the loading.

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I. Introduction

The theory of pseudo-elasticity has been developed by Ogden and Roxburgh [1] and its application by Ogden [2] and Lazopoulos and Ogden [3] in rubber like solids. In this theory, loading is described by an elastic strain-energy function and unloading by a different strain energy function. The strain energy function is affected through incorporation of an additional variable into the strain energy function, which is then referred to as a pseudo-energy function.

In earlier works in pseudo-elasticity, it is assumed that the material response remains isotropic, but in general when residual strain exists, then the material response will be anisotropic. For more general discussion, we refer to Hoger [4] and Johnson and Hoger [5]. Readers can refer to alternative model for Mullins effect and for general deformations in incompressible and compressible materials, see Beatty and Krishnaswamy [6], Krishnaswamy and Beatty [7], respectively.

Since the early work of Rivlin [8], the axial shear problem and pure azimuthal shear for general incompressible materials have received some additional attention by Ogden [9], Jiang and Ogden [10], Kumar and Kumar [11], Horgan and Saccomandi [12], Simmonds and Warne [13], and Kanner and Horgan [14]. The problem for compressible materials has, in fact, been more widely investigated by Jiang and Ogden [10], Jiang and Knowles [15], Polignone and Horgan [16], Jiang and Beatty [17], Horgan and Saccomandi [18], Horgan et al. [19], and Wineman [20].

The theory of pseudo-elasticity developed in [1, 2] is summarized in Sections II and III. We apply the theory, with residual strains allowed for, to a problem involving non-homogeneous deformation. We consider here the pure axial shear of a thick walled circular cylindrical tube of incompressible initially isotropic material. We formulated the boundary value problem for general incompressible pseudo-elastic material for both loading and unloading. The inner circular boundary is taken to be fixed and the outer boundary rotating relative to it at fixed radius by application of a suitable shearing stress.

Sections IV-VII are related to the equations for pure axial shear. Firstly, we have taken care of the neo-Hookean form of strain-energy function. Then the theory of pseudo-elasticity is applied and equations for unloading are obtained in general form. By using a specific example for the constitutive model, we calculate the residual strain explicitly.

In Section VIII, the stress softening effect of the shear stress on unloading is numerically compared with the loading.

II. Pseudo-elasticity Theory

The theory we are considering here is developed in [1, 2] and the theory of pseudo-elasticity adopted is based on the pseudo-energy function $W(F, \eta)$, where F is the deformation gradient relative to the natural configuration of the material and η is the additional variable, which may be active or inactive. If it is active, we set it to the constant value unity and write

$$W_0(F) = W(F, 1), \tag{1}$$

for the resulting strain energy function. When active, it is taken to depend on the deformation gradient and we write $\eta = \hat{\eta}(F)$ and define the strain energy function by

$$w(F) = W(F, \hat{\eta}(F)). \tag{2}$$

For the reference of non-linear elasticity we refer to, for example, Ogden [9] and Holzapfel [21]. In this paper, we consider the material to be incompressible, so that the constraint

$$\det F = 1, \tag{3}$$

is satisfied. The nominal stress tensor associated with the strain energy (1) is denoted by S_0 and is given by

$$S_0 = \frac{\partial W_0}{\partial F}(F) - \rho_0 F^{-1}, \tag{4}$$

where ρ_0 is the Lagrange multiplier arising from the constraint (3).

In [3], a variational procedure was used to show that, where η and F are related, in general implicitly, through the relation

$$\frac{\partial W}{\partial F}(F, \eta) = 0. \tag{5}$$

This equation, which identifies stationary points of $W(F, \eta)$ with respect to η, defines a hypersurface in the 10-dimensional (F, η)-space to which values of η must be restricted, subject to the constraint (3). If (5) defines η uniquely in terms of F, then we may write formally

$$\eta = \hat{\eta}(F). \tag{6}$$

We introduce w for the resulting strain energy function defined by (2).

In view of (5), the nominal stress tensor S associated with $w(F)$ is simply

$$S = \frac{\partial w}{\partial F}(F) - \rho F^{-1} = \frac{\partial W}{\partial F}(F, \eta) - \rho F^{-1}, \tag{7}$$

where the right hand side is evaluated for η given by (6) and ρ is the counterpart of ρ_0, when η is active. Cauchy stress tensors denoted by σ_0 and σ for inactive and active η, respectively, are related to S_0 and S, respectively, by

$$\sigma_0 = FS_0, \quad \sigma = FS. \tag{8}$$

We require η to be an objective scalar variable and that $W(F, \eta)$ satisfies usual objectivity condition

$$W(QF, \eta) = W(F, \eta) \text{ for all proper orthogonal } Q. \tag{9}$$

The requirement of η then ensures that the dependence of η on F determined from (5) is objective, and objectivity of w in (2) then follows.

III. Isotropy

For an isotropic material, we write the pseudo-energy function in the form

$$\overline{W} = (\lambda_1, \lambda_2, \lambda_3, \eta), \tag{10}$$

where $\lambda_1, \lambda_2, \lambda_3,$ the principal stretches of the deformation, are subject to incompressibility condition

$$\lambda_1\lambda_2\lambda_3 = 1. \tag{11}$$

Equation (5) specializes to

$$\frac{\partial \overline{W}}{\partial \eta} = 0, \tag{12}$$

and the principal Cauchy stresses are given by

$$\sigma_i = \lambda_i \frac{\partial \overline{W}}{\partial \lambda_i} - p, \quad i = 1, 2. \tag{13}$$

We now write

$$W(\lambda_1, \lambda_2, \eta) = \overline{W}(\lambda_1, \lambda_2, \lambda_1^{-1}\lambda_2^{-1}, \eta), \tag{14}$$

so that (12) becomes

$$\frac{\partial W}{\partial \eta}(\lambda_1, \lambda_2, \eta) = 0. \tag{15}$$

Accordingly, equation (6) may be represented as

$$\eta = \hat{\eta}(\lambda_1, \lambda_2) = \hat{\eta}(\lambda_2, \lambda_1), \tag{16}$$

the symmetry in λ_1 and λ_2 being noted. Equations in (13) are combined, on elimination of p, to give

$$\sigma_1 = \lambda_1 \frac{\partial \overline{W}}{\partial \lambda_1}, \quad \sigma_2 - \sigma_3 = \lambda_2 \frac{\partial \overline{W}}{\partial \lambda_2}. \tag{17}$$

Note that we are now using the same notation W for the pseudo-energy function as in Section I, but with the dependence now on the two independent stretches in $\lambda_1,\ \lambda_2$ and η. Similarly, the notation for the function $\hat{\eta}$ has been retained.

IV. Pure Axial Shear of a Long Circular Cylindrical Tube

We consider the axial shear of a hollow circular cylindrical tube. The body is composed of an incompressible isotropic hyperelastic material. The tube cross-section is defined by

$$A \leq R \leq B, \quad 0 \leq \Theta \leq 2\pi \tag{18}$$

in polar coordinates (R, Θ) in the reference configuration. Let (r, θ) be the corresponding polar coordinates in the deformed configuration. The deformation investigated is that of pure axial shear described by

$$r = R, \quad \theta = \Theta, \quad z = Z + w(R), \tag{19}$$

where we are now regarding w as a function of $r(= R)$; correspondingly, we write $a = A, b = B$ so that $a \leq r \leq b.$ We also use the notation $v = w'(r)$ and note that the deformation is locally a simple shear with an amount of shear v, the direction of shear being the axial direction.

Thus the stress components are $\sigma_{r\theta} = \sigma_{12}, \quad \sigma_{rz} = \sigma_{13}.$

Hence

$$\sigma_{rz} = \hat{W}_v(v, \eta). \tag{20}$$

The equilibrium equations in absence of body forces reduce to

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0, \tag{21}$$

and

$$\frac{\partial}{\partial r}(r\sigma_{rz}) = 0. \tag{22}$$

On integrating the latter using boundary condition, we get

$$\sigma_{rz} = \frac{b}{r}\tau_0, \tag{23}$$

where $\tau_0 > 0$ is a prescribed constant. Using (20), equation (23) can be written as

$$\hat{W}_v(v, \eta) = \frac{b}{r}\tau_0. \tag{24}$$

This equation can be solved for v and hence, by integration for the displacement function $w(r)$. This equation is the same which arises in the corresponding elastic problem, and for discussion of requirement on the function $\hat{W}_v$ for the equation to yield a solution, we refer to Jiang and Ogden [22]. We impose the boundary conditions

$$w(a) = 0, \quad w(b) = d, \tag{25}$$

on w which correspond to the inner boundary at the tube being held fixed and the outer boundary being displaced axially by the prescribed distance d.

V. Loading

For loading, we take η to be inactive so that equation (24) is written as

$$\hat{W}_0'(v) = \frac{b}{r}\tau. \tag{26}$$

To ensure that this equation yields a unique solution for v as a function of r for all $\tau \geq 0$, we assume that

$$\hat{W}_0''(v) > 0, \quad \hat{W}_0'(v) \to \infty \text{ as } v \to \infty. \tag{27}$$

For incompressible and compressible isotropic elastic materials, the azimuthal shear problem has been considered by several authors and we refer the paper by Jiang and Ogden [22] for complete discussion and reference. Thus we have

$$\hat{W}_0'(v) = \mu 2^{k+1} v(2 + v^2)^{-(k+1)} = \frac{b}{r}\tau \text{ it holds if } k \leq -\frac{1}{2}. \tag{28}$$

For special case $k = -1$,

$$\hat{W}_0'(v) = \mu v = \frac{b}{r}\tau. \tag{29}$$

From (29),

$$v = \frac{b}{\mu r}\tau, \quad w(r) = \frac{b\tau_0}{\mu}\ln\left(\frac{r}{a}\right). \tag{30}$$

The prescribed distance d of the outer surface relative to the inner one is given by the linear relationship between d and τ as

$$d = w(b) = \frac{b\tau_0}{\mu}\ln\left(\frac{b}{a}\right). \tag{31}$$

The total energy, E say, in the material due to the deformation is given by

$$E = \mu\pi\int_a^b v^2 r dr = \frac{\pi b^4 \tau^4}{\mu}\ln\left(\frac{b}{a}\right). \tag{32}$$

Let τ_m be the maximum value of τ reached on loading and let v_m be the corresponding value of v, so that

$$\hat{W}_0'(v_m) = \frac{b}{r}\tau_m. \tag{33}$$

We set

$$\hat{W}_m = \hat{W}_0(v_m), \tag{34}$$

which, of course, depends on r and τ_m. For the neo-Hookean material, we have

$$\hat{W}_m = \frac{1}{2}\mu v_m^2, \quad v_m = \frac{b\tau_0}{\mu}. \tag{35}$$

VI. Unloading

We now consider unloading with η active so that the identity

$$\hat{W}_\eta(v, \eta) = 0 \tag{36}$$

gives η in terms of v. The axial shear (24) is written as

$$\hat{W}_v(v, \eta) = \frac{b}{r}\tau_0, \quad \tau \le \tau_m, \tag{37}$$

which is the same as (26) at $v = v_m$, $\eta = 1$ (for all r).

When $\eta = \hat{\eta}(v)$ is determined from (36), the solution of (37) for v describes the unloading deformation path as τ reduces from τ_m to 0. When $\tau = 0$, we must have

$$\hat{W}_v(v, \eta) = 0, \tag{38}$$

which is solved locally with (36) to give a pair of values (v, η) at each radius r through the tube.

We now illustrate the results for the model as described in [1, 2] which, when specialized for axial shear, gives

$$\hat{W}(v, \eta) = \eta\hat{W}_0(v) + \phi(\eta), \tag{39}$$

and we have

$$\phi'(\eta) = -\hat{W}_0(v), \quad \phi(1) = 0, \quad \phi'(1) = -\hat{W}_m. \tag{40}$$

Equation (37) becomes

$$\eta\hat{W}_0'(v) = \frac{b}{r}\tau, \quad \tau \le \tau_m. \tag{41}$$

Now when $\tau = 0$, either $v = 0$ or $\eta = 0$. When $\eta = 0$, the first equation of (40) gives

$$\hat{W}_0(v) = -\phi'(0), \tag{42}$$

which (locally) determines uniquely the residual values v_r of v.

Since ϕ, and hence $\hat{\eta}(v)$, depends on r through $\hat{W}_m$, the resulting unloading strain energy obtained from (39) is inhomogeneous because it depends explicitly on r as well as on v.

To illustrate the theory further, we now select a specific form of ϕ, such that

$$\phi(\eta) = \frac{1}{4}\mu v_0^2(\eta - 1)^2 - \hat{W}_m(\eta - 1), \tag{43}$$

where v_0 is a positive (dimensionless) material constant.

The first equation of (40) then gives

$$\frac{1}{2}\mu v_0^2(\eta - 1) = \hat{W}_0(v) - \hat{W}_m. \tag{44}$$

For unloading, the pseudo-energy $W(v, \eta)$ becomes the strain energy

$$\hat{W}(v, \hat{\eta}(v)) \equiv \hat{W}_0(v) + (\hat{W}_0(v) - \hat{W}_m)^2/\mu v_0^2, \tag{45}$$

as a function of v, and this shows explicitly its dependence on $\hat{W}_0(v)$ and hence on r.

For the neo-Hookean material, equation (44) becomes

$$\eta = \hat{\eta}(v) \equiv 1 + (v^2 - v_m^2)/v_0^2, \tag{46}$$

and equation (41) becomes

$$[1 + (v^2 - v_m^2)/v_0^2]v = \frac{b\tau}{\mu r}, \quad \tau \le \tau_m. \tag{47}$$

VII. Residual Strains

When $\eta = 0$, we have from (44),

$$\hat{W}_0(v_r) = \hat{W}_m - \frac{1}{2}\mu v_0^2. \tag{48}$$

Now as there is to be a solution of this equation for v_r, we must have

$$\hat{W}_m > \frac{1}{2}\mu v_0^2. \tag{49}$$

If the inequality (49) does not hold, then there is no residual strain and the solution $v = 0$ applies. Thus $\tau = 0$.

Hence we restrict to the neo-Hookean strain energy so that η is given by (46) on unloading and v by (47). Then set $\eta = 0$ and $v = v_r$ in equation (46), and have

$$v_r^2 = v_m^2 - v_0^2. \tag{50}$$

This has a real solution for $v \neq 0$ only if $v_m > v_0$ locally. If $v_m \leq v_0$ for all r, then $v = 0$ is the only solution when $\tau = 0$. Thus there is no residual strain anywhere in the material.

To show the dependence of v_m on r, we write

$$v_m(r) = v_m = \frac{b\tau_m}{\mu r}. \tag{51}$$

Then

$$v_m(a) = \frac{b\tau_m}{\mu a}, \quad v_m(b) = \frac{\tau_m}{\mu}. \tag{52}$$

Now consider the three cases separately.

(1) $v_m(a) \leq v_0$ and $\tau_m \leq \mu v_0 a/b$. In this case, there is no residual strain, $v = 0$ for all r and $w(r) = 0$.

(2) $v_m(b) \leq v_0 \leq v_m(a)$. In this case, r_0 $(a \leq r_0 \leq b)$ is the value of r such that $v_m(r_0) = v_0$. Then

$$v_0 = \frac{b\tau_m}{\mu r_0} = v_m \frac{r}{r_0}, \tag{53}$$

and hence

$$v_m \leq v_0 \text{ for } r_0 \leq r \leq b, \quad v_m \geq v_0 \text{ for } a \leq r \leq r_0. \tag{54}$$

The solution for v corresponding to $\tau = 0$ is treated as a special case:

$$v = 0 \text{ for } r_0 \leq r \leq b, \quad v = v_r = \sqrt{v_m^2 - v_0^2} \text{ for } a \leq r \leq r_0, \tag{55}$$

and from (53), we get v_r in terms of r as

$$v = v_0 \sqrt{r_0^2 - r^2}/r^2. \tag{56}$$

The solution for the residual value of $w(r)$, written as $w_r(r)$, is then

$$w_r(r) = c + v_0 \left[\sqrt{r_0^2 - r^2} - \frac{r_0}{2} \log\left(\frac{r_0 + \sqrt{r_0^2 - r^2}}{r_0 - \sqrt{r_0^2 - r^2}} \right) \right] \text{ for } a \leq r \leq r_0, \tag{57}$$

where c is a constant obtained using boundary condition (18) as

$$c = v_0 \left[\frac{r_0}{2} \log\left(\frac{r_0 + \sqrt{r_0^2 - a^2}}{r_0 - \sqrt{r_0^2 - a^2}} \right) - \sqrt{r_0^2 - a^2} \right], \tag{58}$$

which is precisely the residual value of d.

(3) $v_m(b) \geq v_0$. In this case, there is a residual strain for all values of r. The value of the residual strain is found to be

$$w_r(r) = c + v_0 \left[\sqrt{b^2 - r^2} - \frac{b}{2} \log\left(\frac{b + \sqrt{b^2 - r^2}}{b - \sqrt{b^2 - r^2}} \right) \right] \text{ for } a \leq r \leq b, \tag{59}$$

where the constant c is given by

$$c = v_0 \left[\frac{b}{2} \log\left(\frac{b + \sqrt{b^2 - a^2}}{b - \sqrt{b^2 - a^2}} \right) - \sqrt{b^2 - a^2} \right]. \tag{60}$$

The total residual energy, denoted by E_r, is calculated in all the three cases discussed here (per length in the direction normal to the considered plane) as:

In case (1):

$$E_r = 2\pi\int_a^b \phi(\eta_m)\, r\,dr \text{ or } 2E_r/\pi\mu v_0^2 = \tau_m^2 b^2 (b^2/a^2 - 1)/2\mu^4 v_0^4. \quad (61)$$

In case (2):

$$E_r = 2\pi\int_a^{r_0} \phi(0)\, r\,dr + 2\pi\int_a^b \phi(\eta_m)\, r\,dr$$

or

$$2E_r/\pi\mu v_0^2 = 2r_0^2 \log\left(\frac{r_0}{a}\right) - (r_0^2 - a^2)/2 + r_0^2/2(1 - r_0^2/b^2)\tau_m^2 b^2. \quad (62)$$

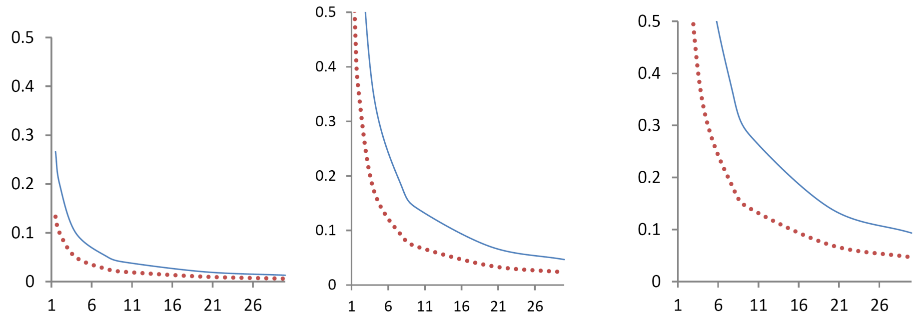
In case (3):

$$E_r = 2\pi\int_a^b \phi(0)\, r\,dr \quad \text{or} \quad 2E_r/\pi\mu v_0^2 = 2r_0^2 \log\left(\frac{b}{a}\right) - (b^2 - a^2)/2. \quad (63)$$

In each case, we can check that $dE_r/d\tau_m > 0$ for the relevant range of values of τ_m.

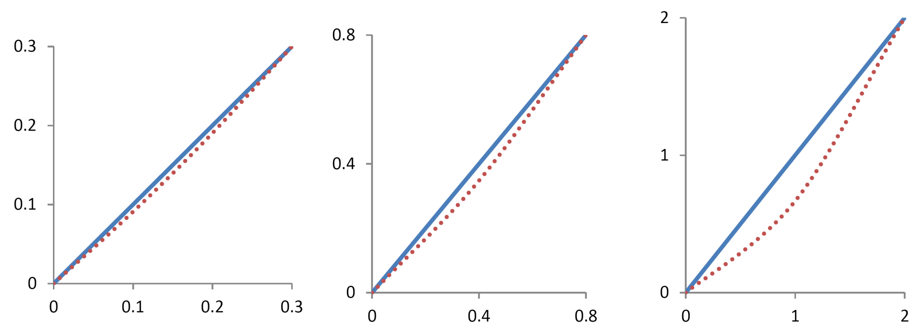
VIII. Numerical Results

In Figure 1, the result for loading is compared with that for unloading at the intermediate point. The upper curve corresponds to unloading and shows that a large deformation is associated with a given stress on unloading as compared with loading. The three cases (1)-(3) in Figure 1 correspond, respectively, to cases discussed in Section VII. It is also observed that the strain decreases monotonically as the radius increases for both loading and unloading.



(1) (2) (3)

Figure 1. Plot of v/v_0 on vertical scale against tube wall thickness $b/a = 2$ on horizontal scale and $\tau/\tau_m = 1/2$ with the following values of $\tau_m/\mu v_0$: (1) 0.2; (2) 0.7; (3) 1.4. The lower curve is for loading and the upper curve is for unloading.



(1) (2) (3)

Figure 2. Plot of unloading shear stress on vertical scale on $r = b$ against $\tau/\mu v_0$ on horizontal scale for $0 \le \tau \le \tau_m$ and tube wall thickness $b/a = 2$ compared with the loading shear stress (straight line) with the values of $\tau_m/\mu v_0$: (1) 0.3; (2) 0.8; (3) 2.

In Figure 2, the stress softening effect is shown for three different values of $\tau_m/\mu v_0$, the shear stress on $r = b$, $\sigma_{0r\theta}$ for loading shown by straight line and $\sigma_{r\theta}$ for unloading. It is observed that the stress softening on loading increases significantly with the extent of loading. It is also observed that the shear stress softening effect increases as r decreases and it is maximum at the inner boundary $r = a$.

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